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Vorticity Transport Integral Concept for Determining Wave Forces on Submerged Bodies

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The vorticity transport integral (VTI) concept affords an analytical procedure that is well suited to determining the wave forces on bodies and structural elements in a marine environment. The concept can be applied to any unsteady flow; however, it is especially suited to the treatment of flows associated with ocean waves in which an object experiences complete flow reversal with wake motion confined to the vicinity of the body. As an integral approach it allows forces to be determined accurately with either an approximate description or an exact solution of the flow. Because the integrals have significant values only near the body, the approach avoids the problems inherent with the evaluation of integrals over large volumes of fluid. The resulting expressions are used in a qualitative appraisal of the various contributions to the total force on a body in an unsteady flowfield.

Introduction

VERY simply stated, the question under study is: How can forces due to the ocean wave interaction with marine structures be estimated to a sufficient degree of certainty? The ocean waves in question can range from well-behaved swells to a complex sea state due to the interaction of currents and large amplitude waves arriving from different directions. The marine structures of interest may vary considerably in size, geometry, and degree of immersion. Structures of particular interest in marine applications are the relatively stationary components of offshore platforms, floating vessels, underwater facilities, marine risers, pipelines, etc.

Our study is based upon the premise that wave forces on structures can be predicted accurately providing descriptions of the far-field kinematics (the ocean wave or sea state) and the near-field kinematics (the fluid-structure interaction) are available. The description ultimately must account for the viscous flow pattern considering flow separation while also accounting for wake sweeping, inclination of the structural elements, and variations of the flow along the length of the structural elements. It is not within the scope of the present study to remark on the various models that have been proposed to account for these near-field phenomena, but, rather, our purpose has been to develop a concept which can be used to 1) explore qualitatively the influences of the various near-field phenomena that have been argued to be significant, 2) compare quantitatively the importance of the phenomena and the model used to describe it, and 3) develop a pragmatic methodology with which forces can be estimated.

Our objective has been to introduce and develop a new concept for determining the hydrodynamic forces on submerged bodies in an unsteady far-field flow. The Vorticity Transport Integral (VTI) concept is an indirect analytical approach that yields a rational basis for calculating forces on the surfaces of a structure due to an unsteady flow. The approach is an indirect approach (as is a control volume approach) in which the forces are expressed as integrals over the unsteady flowfield. Because the resulting integrals are expressed in terms that represent defects in the far-field flow (similar to the momentum integral method of boundary-layer theory) their evaluation is tractable. The approach is capable of taking into account as many details of a transient flow as an investigator is willing to describe. For example, wake sweeping, asymmetry in the flow separation pattern, and other three-dimensional aspects of a time-dependent viscous flow can be considered ultimately.

Although the presentation of the concept for a general flow and system orientation is our goal, this initial exposition is restricted to a time-dependent planar flow about a circular cylinder. The simplicity of a two-dimensional flow allows sufficient generality of the unsteady flow conditions while eliminating unnecessary complexities at this stage of development.

Historical Development

Brief History

The current analytical view of the wave force problem for cylinders evolved from the early work of Stokes[1] who was concerned with the very slow motion of a pendulum. He found the force to be represented by an expression containing at least two terms. One was proportional to the product of the mass of the fluid displaced by the pendulum and the acceleration. This term is referred to as the inertial term. A second term was proportional to the product of the surface area of the pendulum and its velocity. This linear drag term is characteristic of low Reynolds number or Stokes flow.

In subsequent work Rayleigh[2] conjectured that the drag term should scale as the square of the velocity so that it would conform to the empirical form used to describe steady flow. However, he noted that at that time a 3/2 dependency had been proposed by aerodynamicists. Morison[3] advanced a two term expression, which is similar to that of Rayleigh and which has been widely used. The Morison equation was advanced on intuitive arguments and empirical evidence. Numerous experimental investigations have been conducted over the past quarter century to determine the empirical inertial and drag coefficients for the Morison equation. All investigations have been characterized by considerable scatter in correlations of the coefficient data.

Wave Force Problem

The general problem of the analysis or design of a marine structure can be divided into at least three aspects: 1) the characterization of the environment (far-field kinematics), 2) the interaction of the structure with the environment (near-field kinematics and wave force methodology), and 3) the optimization of the structure to achieve the maximum benefit at the least risk. Considerable effort has been expended on the environmental aspect in an attempt to characterize the sea state (wave height, current velocity, water particle kinematics, etc.)

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to which a structure might be subjected. The description of the interaction of the sea with an offshore structure through the wave force methodology predicts the forces and motions of the structure as a function of the sea-state variables. The optimization aspect has received considerable attention; however, progress has suffered from the inadequacy of the environmental characterization and of the wave force methodology. The ultimate goal of an optimum structure for a specified use, life, and location is still in the future.

The difficulties of the environmental aspect have long been recognized as a source of significant uncertainty in design problems. Consequently, this aspect has received deserved attention. Unfortunately, the wave force methodology aspect of the design of offshore structures has not been as astutely appraised. Current practice in the design and analysis of offshore structures and in the analysis of data from scale experiments assumes the validity of the Morison equation. The Morison equation is the currently used algorithm for wave force calculations despite several anomalies. First, a significant scatter in drag and inertial coefficient data has persisted through the decades despite the refinement of experimental techniques. The second anomaly is the ambiguity in using the Morison equation for a realistic sea state and on a skewed cylinder where either a two- or three-dimensional representation is appropriate. Finally, the Morison equation has proven to be inadequate in correctly accounting for loads on complex structures.

The hope has been that, with improved wave kinematic data to characterize the environment either statistically or deterministically and with careful experiments to determine the loads, the Morison equation could be "forced to fit" conditions of interest in design. The improvements in load and environmental data have not been removed or reduced the anomalous behavior. The Morison equation has come to be regarded as an inadequate wave force algorithm for describing wave-structure interaction with sufficient accuracy to attain the confidence level in design and optimization which potentially should be achieved with existing data.

Development of a Wave Force Methodology

A new wave force methodology capable of surmounting the current inadequacies can be achieved through several phases of development. The first or "conceptual phase" involves the exploration of the physical phenomena present in the problem and the review of technologies that can be adapted and transferred to the problem. A fresh and broad view of the problem is important in this phase. Without the concept no further development is possible.

To be fully implemented as a methodology, any concept must pass through phases of development which we term the "formulation phase," the "validation phase" and, finally, the interminable "correlation phase." The formulation phase involves the derivation of analytical expressions or algorithms for a given physical configuration from the concept. Validation consists of using the formulated expressions with experimental data to indicate that the predictions of the methodology are both plausible and consistent. The correlation phase involves the processing of large quantities of experimental data with the methodology to establish relationships between the far-field kinematics and the near-field kinematics.

An initial investigation of a unified wave force methodology was undertaken by the author in 1977[4] as an initial conceptual phase study. One of the concepts from the study is the vorticity transport integral concept which affords a rational approach for formulating an analytical wave force methodology. This paper is intended as a conceptual phase study only.

Vorticity Transport Integral Method

Near-Field Phenomena

The near-field phenomena can be considered to consist of three parts: 1) diffraction, 2) boundary-layer flow, and 3) wake kinematics.

Diffraction is that phenomenon by which energy is transmitted laterally along a wave crest when a portion of a wave train is interrupted by a barrier. The convention is to consider diffraction to be dominant when the body size is comparable to the wavelength. However, diffraction is always present as the process by which the unsteady flow in a wave accommodates to the presence of a body. In short, diffraction determines the inviscid unsteady flowfield around a structure.

The boundary layer and wake phenomena due to the flow around a body are closely coupled. Normally, viscous effects are confined to a thin layer on a body surface, and the fluid in this momentum deficient layer flows into a thin wake sheet aft of the body. This symmetric flow pattern, which is characteristic of a low Reynolds number flow, is dominated by viscous drag. As the Reynolds parameter of the flow increases, the thickness of the boundary layer decreases. When an adverse pressure gradient (due to deceleration in the flowfield) is present, the thickness of the boundary layer tends to increase rapidly as its forward momentum is opposed by both shear and pressure forces. At some point on the body surface, the separation point, the retarding shear and pressure forces cause the flow to break away from the surface and create a recirculating wake. This recirculating region will alter the pressures distribution on both the upstream and downstream portions of the body. The resulting asymmetry causes the familiar drag forces associated with high Reynolds number flows. The separation point is a function of the momentum transported by the boundary layer. The complexity of transition from laminar to turbulent boundary-layer flow also must be considered.

Direct vs Indirect Approach

The forces on a stationary surface of a body that is wholly or partially immersed in an unsteady flow are either normal to the surface and proportional to the pressure or tangential to the surface and proportional to the shear. Therefore, to calculate the total force on a body one may simply compute these two force components on incremental surfaces of the body and sum these forces vectorally over all surface increments. This may be termed a direct approach to computing forces. The direct approach requires an accurate description of the flowfield especially near the body where gradients in the flow properties determine the shear and pressure.

Most frequently one takes an indirect approach to computing forces due to flow around a body. In general, the indirect approach consists of integrating the equations of motion over the flowfield. The resulting expressions will contain integrals of the surface forces that may be replaced by the total force vectors. Such expressions can be rearranged so that the total force is expressed as the sum of integrals over the flowfield. Each of the integrals represents the change of momentum in some portion of the flowfield. Through the device of D'Alembert's principle the integral terms may be thought of as components of the total force, which are frequently called inertial forces, drag forces, wave-making forces, etc.

The most widely used indirect approach for the determination of the total force on a body is the control volume formulation. Usually, certain of the volume integrals are transformed through Green's theorem into surface integrals. The control volume approach is quite expeditious for steady flows in that only the surface integrals must be evaluated. For unsteady flows the usual control volume approach becomes somewhat clumsy and can be easily misinterpreted. However, the indirect approach has a significant advantage in that good accuracy for the integrals can be achieved with an approximate description of the flowfield. For this reason we have proposed an alternate indirect approach which we have termed the Vorticity Transport Integral concept.

Basis of the Concept

The basis for this concept is the vector form of the Navier-Stokes equations for an incompressible viscous fluid. To sim-

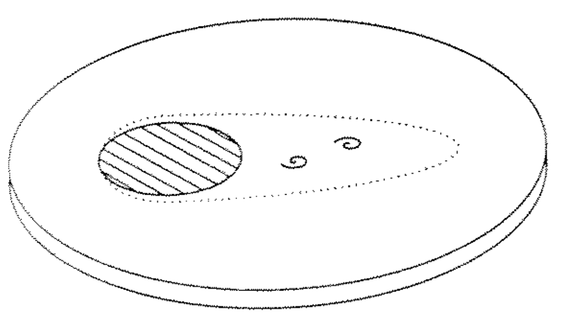

Fig. 1 Two-dimensional planar flow scheme depicting a right circular cylinder (shaded) and the regions of viscous effects; the dotted line encompasses the boundary layer and wake regions.

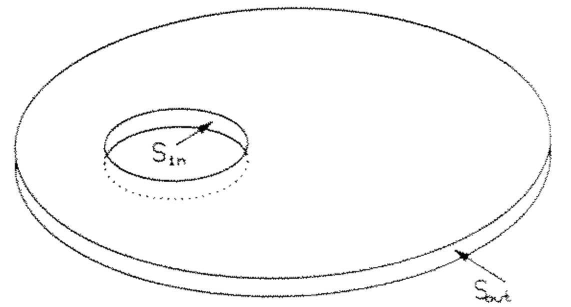

Fig. 2 Control volume which considers the fluid between the surface adjacent to a right circular cylinder (S_{in}) and the surface encompassing all near-field effects (S_{out}).

plify the development the usual body forces have been deleted and only a planar flow is discussed (see Fig. 1). These terms are unnecessary to an initial understanding of the concept, and their inclusion leads to undue complexity at this stage. The usual acceleration operator is replaced by Lagrange's relationship

$$\frac{\partial u}{\partial t} + \omega \times u + \mathrm{grad}\,(q^2/2) \tag{1}$$

where ω is the vorticity vector and $q^2 = u \cdot u$ the square of the magnitude of the velocity vector. Using this relationship the Navier-Stokes equations can be written as

$$\frac{\partial u}{\partial t} + \omega \times u = -\nabla \tilde{p} - \nu \nabla \times \omega \tag{2}$$

where

$$\tilde{p} = p/\rho + q^2/2$$

The Navier-Stokes equations are integrated over an arbitrary volume of fluid surrounding the body upon which the forces are to be determined. The volume integral terms involving pressure and shear may be converted conveniently into surface integrals by Green's transformation theorem. These surface force integrals can be evaluated in two parts: One integral over the surface adjacent to the body, and the other over the remainder of the surface encompassing the fluid (see Fig. 2). This is accomplished as follows:

$$\iiint \nabla \tilde{p}\, \mathrm{d}v = \oint_{S_{out}} n\tilde{p}\,\mathrm{d}s + \oint_{S_{in}} n\tilde{p}\,\mathrm{d}s \tag{3a}$$

$$\iiint \nu \nabla \times \omega\, \mathrm{d}v = \oint_{S_{out}} \nu n \times \omega\,\mathrm{d}s + \oint_{S_{in}} \nu n \times \omega\,\mathrm{d}s \tag{3b}$$

Then the force F on the body is the sum of the shear and pressure forces acting on the surface of the body adjacent to S_{in}, which is

$$F = \oint_{S_{in}} \rho n \tilde{p}\,\mathrm{d}s + \oint_{S_{in}} \mu n \times \omega\,\mathrm{d}s \tag{4}$$

By this device the surface integrals adjacent to the body yield the forces on the body and the integrated equations of motion become

$$F = -\oint_{S_{out}} \rho n \tilde{p}\,\mathrm{d}s - \oint_{S_{out}} \mu n \times \omega\,\mathrm{d}s - \iiint_V \rho \frac{\partial u}{\partial t}\,\mathrm{d}v - \iiint_V \rho\omega \times u\,\mathrm{d}v \tag{5}$$

with n the outwardly directed normal to the surface element $\mathrm{d}s$. Thus, using this expression we can compute the forces on a body by evaluating surface integrals on an arbitrary surface surrounding the body provided the averages of the local acceleration and $\omega \times u$ are known within the volume. The integral expressions presented are comparable to those of a conventional control volume analysis. Needless to say, the evaluation of these integrals over extensive volumes of fluid are formidable and merit a skepticism as to their usefulness.

A significant simplification of the preceding expression results from an identical integration of the far-field description of the flow (the unsteady irrotational flow pattern that, according to convention, would have existed had the body not been present). An integration over a volume identical to that discussed earlier but including the fluid that would have been displaced by the body results in

$$-\oint_{S_{out}} \rho n \tilde{P}\,\mathrm{d}s = \iiint_V \rho \frac{\partial U}{\partial t}\,\mathrm{d}v + \iiint_{V_I} \rho \frac{\partial U}{\partial t}\,\mathrm{d}v \tag{6}$$

where U is the far-field velocity, V the total volume of fluid excluding the body, and V_I the volume displaced by the body. By considering the arbitrary volume of integration to be extensive enough to encompass all near-field effects, i.e.,

$$\oint_{S_{out}} \mu n \times \omega\,\mathrm{d}s = 0 \qquad \tilde{p} = \tilde{P}$$

the combination of Eqs. (5) and (6) yields

$$F = \iiint_{V_I} \rho \frac{\partial U}{\partial t}\,\mathrm{d}v + \iiint_V \rho \frac{\partial}{\partial t}(U-u)\,\mathrm{d}v - \iiint_V \rho(\omega \times u)\,\mathrm{d}v \tag{7}$$

The preceding expression has the distinct advantage over both the conventional control volume expression and Eq. (5) in that the integrands have nonzero values only over the near-field portions of the flow. For the flow outside the near-field region $u \to U$ and $\omega \to 0$.

The foregoing is intended as a conceptual phase development and not as a detailed formulation phase exposition. The formulation and application studies for various flow situations will be the subject of future efforts.

Significance of the Integral Terms

A discussion of the above expressions should begin by recalling that the three integrals in Eq. (7) are all inertial terms that arise from the integration of the acceleration vector over the flowfield. Also, one should note that an explicit drag term is absent.

At first pause, one is tempted to consider the first integral to be the classical inertial term (acceleration evaluated at the centroid of the body times the mass of the fluid displaced by the body) and the second integral to be the virtual mass contribution to the inertial force. It should follow that the third integral would contain the drag contribution. As with many first judgments this appraisal does not stand the test of critical scrutiny.

Let us examine the first integral which is the average of the far-field acceleration over the volume of the body multiplied by the mass of the displaced fluid. Only in the limit of a small body or a negligible spatial variation of the acceleration in the

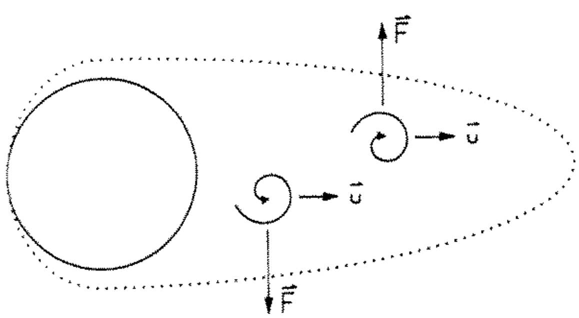

Fig. 3 The force contribution of the vorticity transport term due to the presence of vorticity in the boundary-layer region.

far field is it equivalent to the acceleration evaluated at the centroid of the body. Therefore, the force contribution from this integral is not necessarily in phase with the inertial force component characterized by the Morison equation. It should also be apparent that, since the force may not act through the centroid of the body, moments about the centroid might also exist due to this term.

Next, consider the third integral which contains the vorticity and was argued to represent the drag contribution. One could argue that the vorticity is associated with the wake and the viscous terms of the boundary-layer flow. These two aspects of the flow are always associated with the concept of the "drag" force. Thus, the third integral must be the kernel of the drag force. To demonstrate that this is not the case is quite simple.

First, consider the clockwise vorticity existing in the wake volume associated with the flow around the upper surface of a body and the counter-clockwise vorticity associated with the flow around the lower surface of the body as depicted in Fig. 3. The cross products of the vectors associated with these two vortices and the convective velocity vectors will produce vectors that are generally perpendicular to the flow (the conventional direction of the drag force is parallel to the flow). Furthermore, if the flow is symmetrical, then these clockwise and counterclockwise contributions will sum to zero. Exceptions to this behavior would exist when the vortices are moving askew to the direction of flow (e.g., during the sweeping of the wake back over the body during flow reversals or when the far-field flow is two-dimensional). However, in the case of wake sweeping this effect would be quite transitory.

The contribution to the third integral arising from the boundary-layer region can be resolved into two force components using the components of the velocity in the boundary layer that are tangential and perpendicular to the surface of the body. The dominant component which is proportional to the square of the local velocity arises from the cross product of the vorticity and the tangential component of the velocity vector. This force component is perpendicular to the front surface of the body and would be directed opposite to the usual direction of the drag force (Fig. 4). The second force component which is $N_{Re}^{-1/2}$ times the first component arises from the cross product of the vorticity and the normal component of the velocity vector. This results in a force component that is tangent to the body surface but is opposite in direction to the usual drag force. One can demonstrate that there are higher-order terms arising from the vorticity so that this integral will consist of a minimum of four terms which scale as $U^{1/2}$, U, $U^{3/2}$, and U^2. The coefficients of these terms will be strongly dependent upon the symmetry of the vorticity field.

In summary, the third integral when evaluated over the wake gives rise to lift forces when the flow is asymmetric. When evaluated over the boundary layer this integral also results in negative drag forces. It is quite apparent that the usual drag contribution does not result from this term.

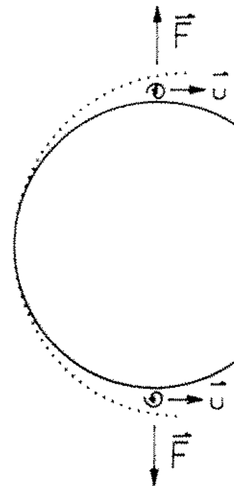

Fig. 4 The force contribution of the vorticity transport term due to the presence of vorticity in the wake region.

Clearly, the positive or balancing portion of the drag force must arise from the second integral in Eq. (7). To explore the diverse properties of this integral, the volume over which it is evaluated will be expressed as the sum of integrals over the wake volume, the boundary-layer volume, the volume influenced by diffraction and reflection, those volumes which represent previous wake vortices, and the remaining volume of unaffected flow.

Considering the wake volume one finds two contributions. The first is the obvious integral of the rate of change of velocity defect $(U-u)$ over the wake volume frozen in time. The second is due to the expansion or contraction of the wake volume with time. Considering steady flow with this formulation, one finds the wake expansion contribution to be the only contribution from the second integral. This contribution represents the "positive" portion of the drag force which opposes the negative contribution from the vorticity integral. For an unsteady flow the expansion of the wake volume is far more complex than for steady flow and will vary with the far-field history. With this realization one easily "sees" the problem with the definition of "drag force" in an unsteady flow.

The preceding discussion illustrates some of the qualitative results that can be gleaned from the vorticity transport integral concept without reference to a specific description of the flow. An illustrative example on the quantitative level is quite desirable at this point but is beyond the limits imposed by the intent and the space allocations for this paper. Thus, exploration of integral expressions for a meaningful description of the unsteady flow that is representative of conditions producing wave forces on a submerged body must be deferred to a later paper. However, assurance that these recondite expressions are capable of yielding conventional results can be established through the simplest example of an unsteady flow—the impulsive start of a viscous flow about a circular cylinder.

Illustrative Example

As the simplest illustration of the application of the VTI expression, consider the growth of the wake volume after the impulsive start of flow past a cylinder and the resulting force in the direction of flow. The example will be limited to the behavior of the flow after a steady-state condition has been reached for the far-field flow. For this condition the first integral of Eq. (7) is zero. The third integral contribution has been discussed in the previous section: integration over the symmetric nonspreading wake yields no force contribution in the direction of flow and integration over the forebody boundary

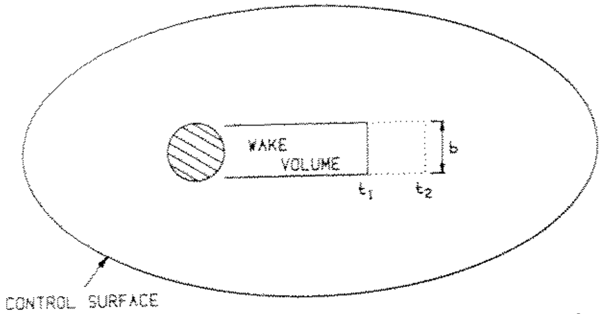

Fig. 5 Expansion of the wake volume of width b from time t_1 to time t_2 after the impulsive start of flow past a circular cylinder.

layer yields a negative drag force which, though important, will not be discussed further.

The object of our attention is the second integral in Eq. (7) and our objective is to illustrate how the positive drag term arises from this integral. The simple model for describing the flow for this illustration consists of the irrotational flow about the cylinder, a thin boundary layer on the forebody, and an elongating wake volume extending from the rear surface of the cylinder. Since the control volume V is fixed in time the second integral in Eq. (7) can be written as

$$F_{II}=\iiint_V \rho\frac{\partial}{\partial t}(U-u)\,\mathrm{d}v=\frac{\partial}{\partial t}\iiint_V \rho(U-u)\,\mathrm{d}v \tag{8}$$

The control volume can be considered to consist of two volumes (Fig. 5): 1) the volume of the wake and 2) the volume outside the wake region. Although there is a boundary-layer volume also present, the temporal derivative of the average velocity defect over this limited region will be neglected. The change in value of the integral between times t_1 and t_2 is due to an extension of the wake volume by an amount ($b\mathrm{d}x$). Thus the value of the change of the integral over the entire volume in Eq. (8) is given by the changes that are associated with the volume ($b\mathrm{d}x$) and represents the difference in the average velocity defect that exists in this region of wake growth from time t_1 to time t_2. Evaluating Eq. (8) for this case, the component of force in the x direction is

$$F_{II_x}=LDU^2\left(\frac{\bar{u}_d}{U}-\frac{\bar{u}_w}{U}\right)_x\frac{b}{D}\frac{c}{U} \tag{9}$$

where L is the length of the cylinder, D is the cylinder diameter, c is the velocity of the wake boundary ($\mathrm{d}x/\mathrm{d}t$), b is the width of the wake volume, and $\bar{u}_d$ and $\bar{u}_w$ are the average local velocities in the region of wake growth due to the inviscid fluid-body interaction and wake effects, respectively. The assumption that the wake grows only in the x direction (parallel to the flow) has been used to simplify the example. This assumption is not necessary but is sufficient for a qualitative illustration of the role of the second integral term. The form of Eq. (9) is identical to that of the classical drag force for a high Reynolds number steady flow where the wake volume contribution to C_D is

$$C_D=2\left(\frac{\bar{u}_d}{U}-\frac{\bar{u}_w}{U}\right)_x\frac{b}{D}\frac{c}{U} \tag{10}$$

Also, as the downstream end of the wake moves away from the vicinity of the body u_d-U which simplifies Eq. (9) somewhat. The contribution, then, from the second integral term of Eq. (7) represents the "positive" portion of the drag force which opposes the negative contribution from the vorticity integral. For a general unsteady flow the expansion of the wake volume is far more complex and will vary with the far-field history.

With the preceding realization one easily sees the problem with the definition of drag force in an unsteady flow. The problem becomes even more difficult when one considers other "volume contributions," spatial variations in the far-field flow, and three-dimensional flow schemes.

Importance of the Methodology

The next question is: If the drag and inertia classification of forces is not meaningful, then what methodology should be used? The Vorticity Transport Integral methodology yields several answers to this question. First, one can use the integrals from which the terms arise to classify the force contributions. The first integral is a far-field inertial contribution, the second integral is an inertial defect contribution, and the third integral yields a vorticity transport contribution. Further classification based upon the physical phenomena present is also clearly indicated; the second and third integrals can be divided into integrals over volumes containing the diffraction region, the boundary-layer region, the wake region, etc. This would enable an investigator to determine the significance of each of the volume contributions to the total force for a specific flow scheme.

Clearly, the real power of the vorticity transport integral methodology will be the capacity to supply the level of either quantitative or qualitative understanding which an investigator desires. By using detailed classifications such as those just outlined, one can map out parametrically the range of dominant terms. Because this is an indirect or integral approach, accurate results can be achieved with a modest effort in representing the details of the near- and far-field kinematics. The concept can easily be extended so that differential contributions to the force on a body can be computed. Also, the relative motion of the surface of a system may be considered. Virtually any marine application could be treated when this concept is fully implemented as a methodology.

Conclusions

The purpose of this study has been to explore the plausibility of developing a unified wave force theory. A unified theory can be defined as an approach that can predict forces with a desired accuracy over the full range of flow parameters. Such a theory should include a rational representation of the wave kinematics containing dispersive and diffractive factors to the degree that they are an important part of any real wave-structure interaction. The theory must also present an adequate representation of transient viscous flow, considering wake sweeping, asymmetry in the separation pattern of the flow, and three-dimensional effects.

The Vorticity Transport Integral (VTI) concept has been explored and found to afford a meaningful approach to formulating a unified analytical wave force methodology. The VTI method is a rational indirect analytical approach to calculating forces on the surface of a structure and is capable of taking into consideration as many details of a transient flow as an investigator is willing to describe. It is an indirect approach in that it consists of integrating the equations of motion over a flowfield that encompasses all near-field effects. If the volume of the flowfield is separated into isolated near-field regions (boundary layer, wake, etc.), one can evaluate the contribution of each to the total force. Thus, through the use of simple flow models, a comprehensive understanding of the contributions of the near- and far-field phenomena to the force can be achieved.

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