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THE IDENTIFICATION OF THE QUADRATIC SYSTEM RELATING
CROSS-FLOW AND IN-LINE, VORTEX-INDUCED VIBRATION

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ABSTRACT

This paper presents an application of the multiple regression method to the identification of the nonlinear relationship between cross-flow and in-line, vortex-induced vibration. Previous results of bispectral analysis of the Castine data by Jong [7] indicated that cross-flow and in-line response are correlated quadratically for both lock-in and non-lock-in cases. Therefore, a second order nonlinear system was used to model the relationship between cross-flow and in-line vibration. The cross-flow response is treated as the input to the nonlinear system and the in-line response is defined as the output. Both time domain and frequency domain multiple regression methods are presented in the evaluation of the quadratic system function under lock-in and non-lock-in conditions respectively. Nonlinear input/output correlations higher than second order in the relationship are shown to be negligible.

NOMENCLATURE

$x(t)$ cross-flow acceleration
 $y(t)$ in-line acceleration
 $y_1(t)$ output from the Case 1 square law operator
 $y_2(t)$ output from the Case 2 square law operator
 $y_s(t)$ simulated in-line response
 y_o d.c. component of the in-line response
 $n(t)$ noise
 $g(u,v), g_1(\), g_2(\)$ second order impulse response function
 $h(u)$ linear impulse response function
 $G(w)$ special form of $g(u,v)$
 K order of linear convolution
 M order of the second order convolution
 $G(W_1, W_2), G_1(W_1, W_2), G_2(W_1, W_2)$ Fourier transforms of $g, g_1,$ and g_2
 $H(w), H_1(W), H_2(W)$ Fourier transform of $h(u), h_1(w)h_2(u)$
MSE mean square error
 $E[]$ expected value operator
 $S_{xx}(W)$ auto spectrum of $x(t)$
 $S_{xy}(W)$ cross spectrum of $x(t)$ and $y(t)$

$B_{xxx}(W_p, W_q)$ auto bispectrum of $x(t)$
 $B_{xxy}(W_m, W_n)$ cross bispectrum of $y(t)$ and $x(t)$
 W frequency (radians/sec)
 $\delta(\)$ delta function
 \underline{I} unity vector
 $\underline{\quad}$ underscore indicates a vector
 $\underline{\quad}]$ square brackets indicate a matrix

INTRODUCTION

Marine risers, pipelines, and hydrophone cables are all examples of structures subjected to vortex-induced vibration. The response of the cylinder depends on a complex interaction between the natural modes of the vibration and the vortex-shedding process. The implementation of good design procedures that account for strumming vibration is becoming more essential as the offshore industry moves into deeper water.

In a spatially uniform flow, lock-in may occur when the vortex-shedding frequency is within a few percent of a cylinder's natural frequency. Sustained periodic vibration results in both in-line and cross-flow directions. The cross-flow motion is dominated by one mode at the natural frequency of the cylinder. The in-line motion is dominated by a frequency twice that of the cross-flow motion. Typical in-line amplitudes are one-half that of the cross-flow displacement [8].

When the shedding frequency is outside the lock-in bandwidth, non-lock-in occurs and the response time histories in both in-line and cross-flow directions are best described as random processes. Several modes may respond in both directions. The cross-flow response frequencies are generally dominated by natural frequencies of the cylinders. The response frequencies typical of the in-line motion are not typically natural frequencies, but are most closely associated with sums of frequencies of dominant cross flow spectral peaks. The evidence of frequency doubling and summing under lock-in and non-lock-in conditions supports the hypothesis that in-line and cross-flow response are non-linearly correlated. As an initial test a bispectral analysis of flow induced vibration data obtained during a field test at Castine, Maine, was performed by Jong [7]. A very clear nonlinear correlation was evident between in-line and cross-flow vibration. The cross-bicoherence provided conclusive evidence that cross-flow and in-line response are correlated