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Prediction of the Dynamic Response of Risers and Cables

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INTRODUCTION

The problem associated with vortex-induced vibration has proved to be extremely difficult, both theoretically and experimentally, due to the complex nature of the fluid-structure interaction mechanism. Most of the key developments of the past decade have resulted from experimental investigations. The emphasis has been primarily on the study of the vortex-shedding problem in the cross-flow direction. However, substantial vibration usually also occurs in the in-line direction but is much less understood. For example, a peculiar experimental observation deduced from the Castine data, is that the in-line response frequencies do not always correspond to the structure natural frequencies; instead they are equal to the double or sum of some of the cross-flow response frequencies for both lock-in and non-lock-in cases. This observation implied that there might exist an identifiable non-linear relationship between in-line and cross-flow motion. This would prove useful in fatigue life estimation, where it is necessary to calculate at specific locations the total stress statistics, caused by both in-line and cross-flow motion. The correlation between the response time series associated with the two directions plays an important role in calculating the total stress statistics. However, no attempt has ever been made to investigate the statistical relationship between in-line and cross-flow responses, or equivalently, drag and lift forces under non-lock-in conditions. Even the answer to the preliminary question of whether they are correlated or independent has not been available.

One purpose of this research was to study the relationship between in-line and cross-flow responses by analyzing the Castine experimental data. Experiments were performed on long flexible cylinders 75 feet in length which were exposed to a uniform current ranging from 0 to 2.4 feet/second. Measurements taken included current, drag, tension and biaxial accelerations at seven locations unequally spaced along the test cylinder.

The acceleration data were double integrated first to obtain the displacement time histories. By a least squares error minimization method, it was possible to evaluate the individual modal displacements contributing to the total responses. Total vibration energies in both directions were then calculated from the modal displacements. The results showed that there exists a strong relation between the total vibration energy in each direction rather than the local behavior in these two directions. This gave a preliminary indication of the existence of a relationship between them.

Linear spectral analysis of the non-lock-in random response data indicated that the responses in the two directions were almost linearly independent. Therefore, a nonlinear relationship was expected. The frequency doubling or summing phenomenon suggested that a higher order spectral analysis was required to investigate whatever nonlinear wave interaction mechanism that might exist.

Bispectrum Analysis and Quadratic System Identification

The bispectrum, which is an ensemble average of a product of three spectral components, was used to detect quadratic correlation between in-line and cross-flow response. Response data of four different cases were analyzed: lock-in, near lock-in, non-lock-in, and shear flow response data from the Arctic test conducted by Vandiver and Kim in 1983. The resulting cross-bicoherence spectrum showed that all vibration at frequencies resulting from doubling and summing were correlated quadratically for all of the four different cases. These bispectral analysis results provided the first convincing evidence of the existence of a quadratic correlation between in-line and cross-flow vibration.

A second order system identification was performed for both lock-in and non-lock-in cases. A second order nonlinear system was used to model the relationship between in-line and cross-flow responses. A residuale error term was introduced to represent any error associated with the imperfection of the model which might be due to the existence of higher order nonlinearities. The resulting residual obtained from the quadratic system identification showed that nonlinearities higher than second order were negligible for both lock-in and non-lock-in cases. The results also showed that in-line and cross-flow responses were linearly independent at non-lock-in, while some linear correlation exists under lock-in conditions.

This quadratic system identification research is continuing and will be written up as part of a Ph.D. dissertation in the next few months.

The Wake Oscillator Model for the Non-Lock-In Case

The wake oscillator model assumes the fluid-structure system is governed by two cross-coupled equations: (1) van der Pol equation of the lift oscillator. (2) equation of motion of the structural oscillator. The wake oscillator model has the following properties: (1) The oscillations are self-excited and self-limiting. (2) The natural frequency of the lift oscillator is equal to the Strouhal frequency fs=StV/D. (3) The wake oscillator has the lock-in or frequency entrainment property. At lock-in, the cross-flow response and lift force can be well approximated by a deterministic sinusoidal time series. The wake oscillator has been used to predict the lift coefficient and cylinder response by Griffin, Currie and Blevins for the deterministic lock-in case.

At non-lock-in, the cylinder response is a random process. The histogram of the cross-flow response from the Castine data reveals no significant departure from the histogram of a Gaussian distribution. A chi-square goodness-of-fit test with a 5% significance level was performed from which it was concluded that the non-lock-in cross-flow response can be approximated by a Gaussian random process.

One purpose of this research was to develop a predictor of the mean-square statistics of the lift coefficient under non-lock-in conditions, for the wake oscillator model. The model is nonlinear, and therefore normal spectral analysis techniques can not be used. Instead, the approximation technique of Gaussian closure, described by Iyengar and Dash, has been employed. The approximation in this technique involves the evaluation of higher statistical moments in terms of second moments, even though the processes are not strictly Gaussian. The cross-flow response is reasonably well modeled as a Gaussian process as mentioned previously, but the lift coefficient is not necessarily Gaussian because it is related to the response by a nonlinear model. It is expected, however, that the lift coefficient would be nearly Gaussian if the damping term in the van der Pol oscillator is weakly nonlinear. Therefore, it would seem to be a very good approximation in this work to use the Gaussian closure technique.

The method by which the mean-square properties of the lift coefficient can be estimated involve the following five steps:

1. Derive a relationship involving the auto- and cross-correlation functions of the response $x(t)$ and lift coefficient $c(t)$, in the van der Pol oscillator model. This

relationship can be derived by multiplying all terms of the van der Pol equation, expressed at time t, by $x(t+\tau)$ and $c(t+\tau)$ respectively, and taking the expected value of all the products. Then assuming Gaussian properties, decompose all higher moments into second moments as they appear in the auto- and cross-correlation functions.

2. Apply the Fourier transform operator to all terms of the relationship in order to replace the auto- and cross-correlation functions with auto- and cross-spectra.

3. Relate the response spectrum Sxx(f) and lift coefficient spectrum Scc(f) with an equivalent transfer function Hv(f) as: $Scc(f)=Sxx(f)\left|Hv(f)\right|^2$. Hv(f) is a function of the coefficients of the van der Pol equations as well as the mean-square value of the lift coefficient, Rcc(0).

4. Relate the response spectrum Sxx(f) and lift coefficientn spectrum Scc(f) by the linear transfer function, He(f), from the equation of motion as; $Sxx(f)=Scc(f)\left|He(f)\right|^2$. He(f) is a function of the structure parameters only.

5. The response and lift coefficient are assumed to be a narrow band random processes with center frequencies at the Strouhal frequency fs=SV/D. The linear transfer function He(fs) is then related to the equivalent transfer function Hv(fs) at the center frequency fs. As a result, it is possible to obtain the mean-square value of the lift coefficient which controls the equivalent damping in the

transfer function Hv(f).

The lift oscillator portion of the research is largely completed and will be written up during the summer of 1984.