APPLICATION OF THE RANDOM DECREMENT TECHNIQUE IN THE DETECTION OF AN INDUCED CRACK ON AN OFFSHORE PLATFORM MODEL

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ABSTRACT

The purpose of this investigation is to apply the Random Decrement Technique to detect induced cracks on an offshore platform model.

The Random Decrement method analyzes the measured output of a system subjected to some ambient random input. After analysis a signal results that is the free vibration response or signature of the mechanical structure. This signal is independent of the input and represents the particular structure tested. The ability to obtain unique response signatures enables one to detect early damage before overall structural integrity is affected. Local flaws, such as cracks, too small to affect the overall structural integrity have a significant effect on the signatures of the higher modes. As a flaw grows, progressively lower modes are affected until overall failure occurs. Damage is detected by studying and comparing the signatures of the higher structural modes.

The tower model is a welded-steel space frame with four primary legs, braced with horizontal and diagonal members. It was fixed to a seismic shaking table which was controlled to provide different loading to the structure such as random, seismic, etc. The tests were performed at the Ecole Polytechnique Federale de Lausanne A systematic study of the effect of structural damage to the Random Decrement Signature was conducted. Responses at various positions along the structure subjected to random input were obtained. Thin saw cuts were induced to a position near the welded joint of the structure. An initial crack depth 1/8 inch was made with a saw blade. Nine additional saw cuts were subsequently made to extend the crack depth to a total break through of the beam. The response time histories were recorded and analyzed to obtain the Random Decrement signatures. From these signatures their sensitivity to the induced cracks at various sizes were correlated. This investigation has shown that the Random Decrement Technique can be used to detect cracks in complex offshore structures.

A finite element space frame model of the structure was developed using the GIFTS and the NASTRAN computer programs. The horizontal and the diagonal braces were modeled by beam elements, the top plate was modeled by a plate element which allows bending and membrane flexibility.

The cracks of the experimental model were reproduced on the computer model and the changes on the dynamic response of the model is investigated.

The work reported by this paper is part of a major effort, which was initiated by the U. S. Office of Geological Survey and the Office of Naval Research to develop better and more reliable techniques for the detection of incipient failure of offshore structures. As a part of this effort several large size models of offshore platforms (1:14 scale) and joints (1:2.5) have been constructed and are being tested. Results of these tests will be reported in subsequent papers. This work was also partially supported by the Swiss National Science Foundation and by the Ecole Polytechnique Federale de Lausanne, Switzerland.

1. RANDOM DECREMENT TECHNIQUE

The Random Decrement Technique was originally developed by Mr. H. A. Cole for the measurement of damping and for the detection of structural deterioration of airplane wings subjected to wind flutter excitation (1, 2). Other applications have then been studied by various other authors (3, 4, 5).

In this section we present a brief, rather intuitive explanation of the principles of Randomdec. A more extensive mathematical derivation was developed in (3) and in Appendix A.

<u>Random Decrement Signature</u>

The response $x(t)$ of a linear system is governed by the following basic equation:

$$m\,\ddot{x}(t) + c\cdot\dot{x}(t) + k\,x(t) = f(t) \qquad (1)$$

The solution of this differential equation depends on its initial conditions and the excitation $f(t)$. Since, for linear systems the superposition law applies, the response can be decomposed into three parts: response due to initial displacement $x_d(t)$, response due to initial velocity $x_f(t)$ and finally the response due to the forcing function $x_f(t)$.

The Randomdec analysis consists of averaging N segments of the length τ_1 of the system response in the following manner: the starting time t_i of each segment is selected such that $x_i(t_i) = x_s$ = constant and the slope $\dot{x}_i(t_i)$ is alternating positive and negative. This process can be represented in mathematical form:

$$\delta(\tau) = \frac{1}{N}\sum_{i=1} x_i(t_i + \tau) \qquad (2)$$

$$\begin{aligned} \text{where } x_i(t_i) &= x_s & i &= 1, 2, 3 \ldots. \\ \dot{x}_i(t_i) &= \geq 0 & i &= 1, 3, 5 \ldots. \\ \dot{x}_i(t_i) &= \leq 0 & i &= 2, 4, 6 \ldots. \end{aligned}$$

The function $\delta(\tau)$ is called the Randomdec signature and is only defined in the time interval $0 \leq \tau \leq \tau_1$. The meaning of the Randomdec signature can now be determined. If the parts due to initial velocity are averaged together, they cancel out because alternately parts with positive and negative initial slopes are taken and their distribution is random. Furthermore, if the parts due to the excitation are averaged they also vanish because, by definition, the excitation is random. Finally only the parts due to initial displacement are left and their average is the Randomdec signature representing the free vibration decay curve of the system due to an initial displacement, which corresponds to the bias level x_s. (Fig 1)

In reality the Randomdec computer converts each segment into digital form and adds it to the previous segments (Fig 2); the average is then stored in the memory and can be displayed on a screen. The number of segments to be averaged for the Randomdec signature depends on the signal shape, usually 400 to 500 averages are sufficient to produce a repeatable signature.

One particularly interesting characteristic of Randomdec technique should be mentioned: it requires no knowledge of the excitation f(t) as long as it is random. Neither the type nor the intensity of the input affect the signature.

Signature Analysis

The procedure for Randomdec analysis of a structure is to establish a reference signature for the undamged system and to compare it with signatures from later recordings.

The response signal x(t) is passed through a broadband filter before being Randomdec processed. As soon as any significant deterioration of the system is developed the signature will change its shape. The signature represents, for broad band filters, a superposition of all the modes inside the filter limits and has a rather complex shape.

In the case of narrow band filtering with only one mode inside the filter limits, there are only single values of frequency and damping need to be considered, since $\delta(\tau = 0) = x_s$ and $\dot{\delta}(\tau = 0) = 0$ always remain the same. In most cases any structural deterioration affects both frequency and damping and gives little indication of the type of structural damage.

Typical Randomdec signatures of response signals passed through a wide band filter and a narrow band filter are shown in Figure 3.

Signature Interpretation

If a structure is being Randomdec analyzed and a change in the signature with respect to the original signature is detected, it is, in many cases, not a priori clear, what kind of structural variation it has to be related to. Many reasons can cause a signature change. We will state only the most significant ones, that have some practical application:

Many steel structures are exposed to natural environmental conditions and have to fight against corrosion, which can be the cause of a considerable reduction of the cross-section. The resulting loss of mass and the decreased stiffness will yield in a different vibrational behavior and result in a changed signature. While the reduced mass will increase the signature frequency, the smaller stiffness tends to lower it.

Another structural deterioration occurring to cyclically loaded structures are the fatigue cracks. This reduces the stiffness and changes the internal damping. Consequently the signature frequency will go down and the internal damping will increase.

Non-welded joints often lose part of their stiffness with loose bolts or damaged rivets, which, in effect, gives similar results as a fatigue crack.

A change in the boundary conditions of a structure can lead to dramatic changes of the Randomdec signature, because of its effects on the stiffness values.

Furthermore, an element subjected to a normal force will also change the bending stiffness (6, 7). It will, for the compressive forces, reduce the stiffness, while a tensile force will stiffen the member. Although this effect is rather small for low forces, it becomes highly important as the load gets close to the buckling load.

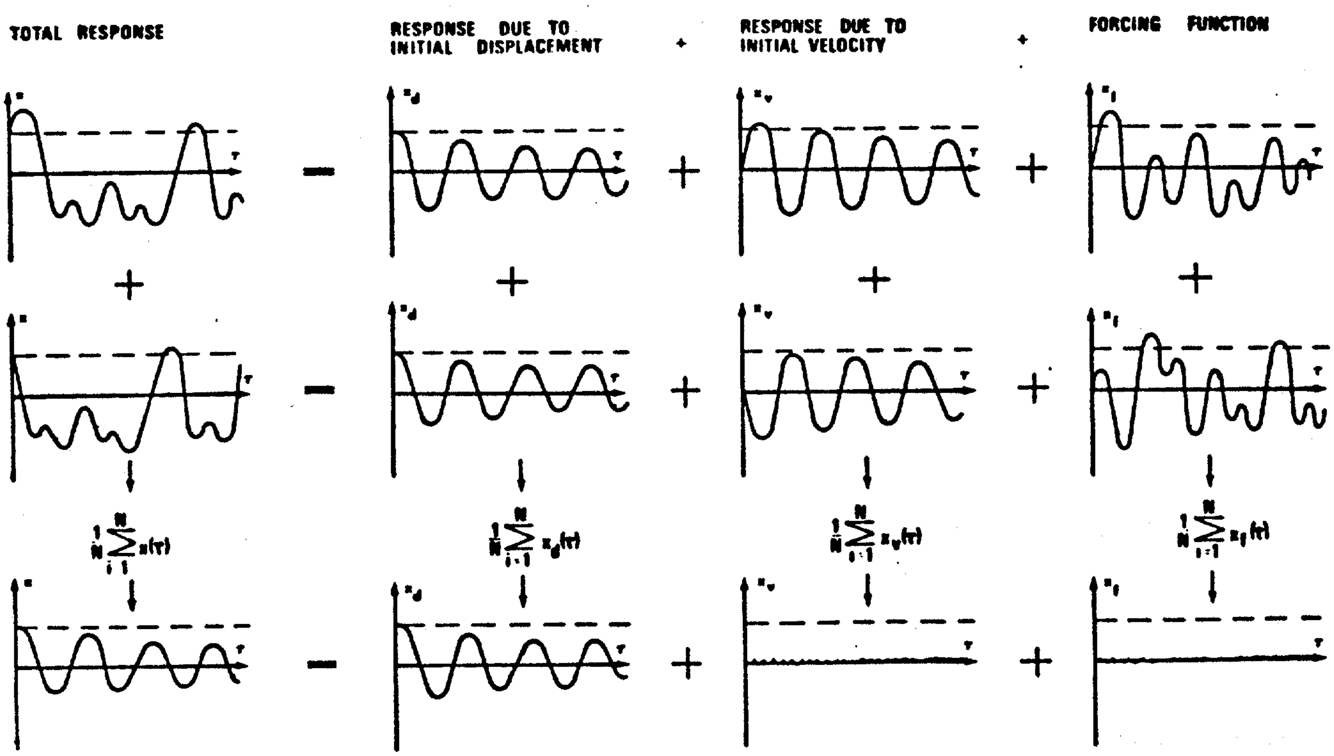


Figure 1: Principles of Randomdec Technique

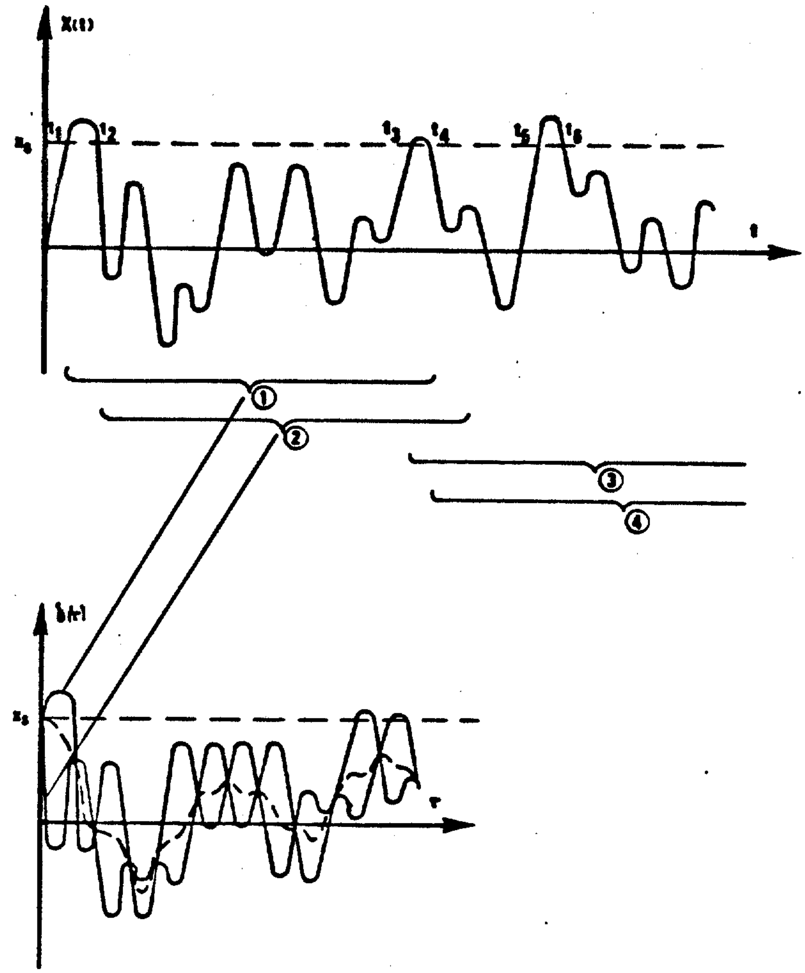


Figure 2: Extraction of the Randomdec Signature

Finally, welded elements often have very high residual stresses, which can reach the yield stress. Any change in the state of residual stresses will then also cause a change in the stiffness and can be detected by the Randomdec signature.

This rather incomplete list shows that Randomdec is very versatile and has many useful applications. On the other hand, the analysis is often complicated by the fact that a structural change affects in most cases more than one of the three characteristical terms m, c, and k. There is no difficulty to detect the change, but it can be rather complex to find out what and how much change has occurred. As far as fatigue cracks are concerned only k and c are affected, where c has in general a negligible influence on the signature frequency. In this case a relation between frequency and stiffness can be set up, which makes Randomdec especially useful for crack detection.

2. FAILURE DETECTION

From the foregoing section it was shown that the Randomdec signature gives a curve which is related to the free vibration decay of the structure with an initial displacement. The scale and form of this curve is always the same even when the intensity of the ambient random forces changes in contrast to spectral density and autocorrelation which vary with changes in the ambient random forces. In this section, the hypothesis and application of the method to failure detection is developed.

A typical experimental setup is shown on figure 4. It should be noted that the spectral analyzer provides a broad view of the location of structural modes which may be used as an aid to specifying filtering requirements. Let us consider now what happens to the signature when a fatigue crack develops in a structure. A fatigue crack introduces additional degrees of freedom which are excited by the random forces. When the crack is small, small blips would show up in the hashy, high-modal density region of the spectral density; in this form detection would be difficult. As the flaw grows, the frequency of the failure mode would be expected to decrease until it approaches the fundamental modes. By the time a flaw reaches the low-frequency range it would be imminent. To detect the failure mode it needs to be interepted at a high enough frequency so that correctivaction can be taken and complete failure avoided. To do this the random signal is passed through a band-pass filter which is set at a high frequency. With the undamped structure, standard Randomdec signatures are established for all loading conditions and environments. If a failure develops, it will have a powerful effect on the signature because it will dynamically couple with structural modes within the bandpass frequencies of the filter. Some fundamental studies were made by J. C. S.

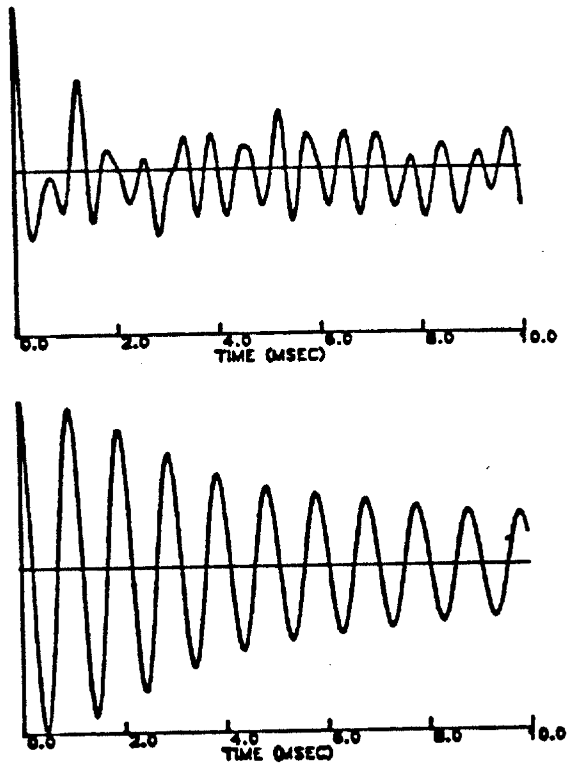
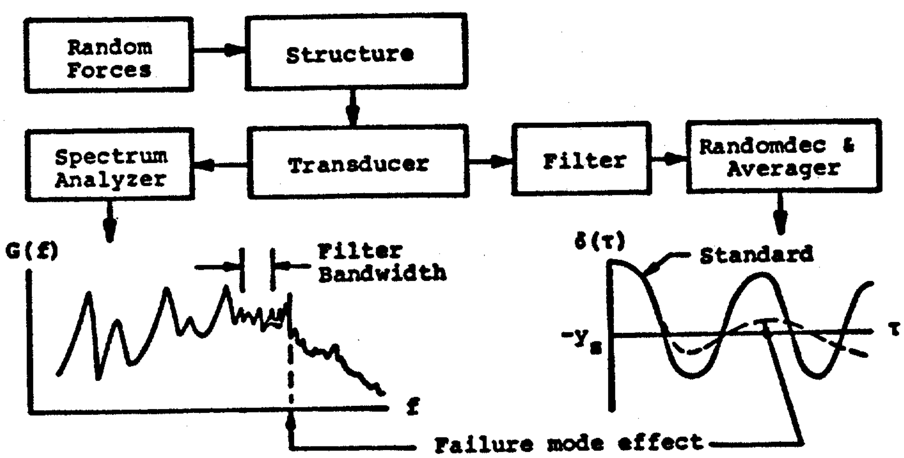


Figure 3: Typical Randomdec Signatures of response signals passed through a wide band filter (above) and a narrow-band filter (below)



Hypothesis:

(1) Flaw introduces additional degree of freedom.

(2) Frequency of flaw mode decreases as flaw size grows.

(3) Flaw mode causes change in signature by:

(a) Dynamic coupling with modes in filter bandwidth.

(b) Nonlinear coupling at subharmonic frequencies.

(c) Friction damping.

Figure 4: Hypothesis on the sensitivity of random decrement signatures to flaws

Yang (see reference 5, 8). For the failure detector, once the standards have been established only parts of the signature at peaks need to be recalculated with warning devices sensitive to voltage changes in the peak values.

A procedure for failure detection is outlined on figure 5, which shows only a single peak for illustration. The standard signature region is first established to a confidence level consistent with percent of false alarms which could be tolerated. For the 95-percent confidence level shown, of course, false warnings would occur 5 percent of the time. Detection would be as shown on the figure. The check on standard deviation, $\overline{\sigma}$, is to prevent false indications due to extraneous input sources other than the normal random excitation, i.e., a sinusoidal force or signal in the electronics. For example, if a sinusoidal force was applied to the structure, the signature would become an undamped cosine wave and fall outside the standard region, but the standard deviation woulf fall to zero. In this case the amber light would go on.

3. DETECTION OF CRACKS IN AN OIL PLATFORM

Experimental Test

A laboratory experiment was conducted to check the sensitivity of the Randomdec signature in detecting

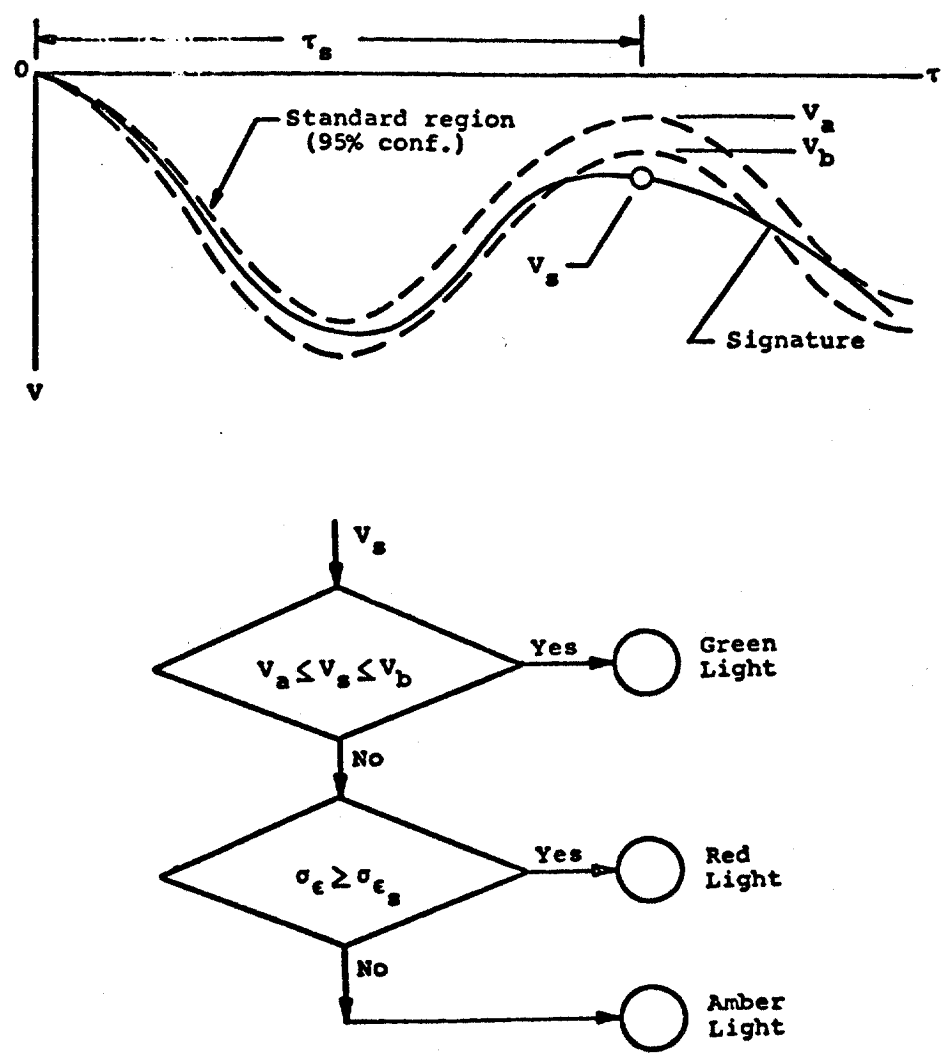


Figure 5: On-line failure detection at a single point on the signature

structural damage. A model oil platform was constructed. It is a welded-steel space frame with four primary legs, braced with horizontal and diagonal members. The primary legs of the model had a diameter of 25 mm (0.984 inch), and the horizontal and diagonal members had diameter of 15 mm (0.591 inch). The structure, its design and the dimensions of all the components are shown in figure 6.

The tower model was fixed to a seismic shaking table which was controlled to provide different loading to the structure such as random, seismic, etc. see figure 7. A systematic study of the effect of structural damage to the Randomdec signature was conducted. Responses at various positions along the structure subjected to seismic inputs from tapes of the El Centro Earthquake were obtained with a number of accelerometers at various intervals of loading, see figure 8.

Saw cuts were induced to the welded section of the structure on the cross beam joining the primary leg of the platform. An initial crack depth of 1/8 inch was made with a saw blade which was approximately 0.07874 cm (0.031 inch) wide. Nine additional saw cuts were subsequently made to extend the crack depth to a toal breakthrough of the beam. At each additional cut, the depth of the notch was increased approximately by 1/16 inch (1.59 mm).

The response time histories at each saw cut depth were recorded and analyzed. Randomdec signatures were obtained for frequency between 4000 Hz and 8000 Hz using bandpass filters. A number of Randomdec signatures were obtained for each saw cut depth in order to confirm the repeatability of the Randomdec signatures. The repeatability of the Randomdec signatures were excellent. This is very essential for the success of the Random Decrement technique in the detection of flaws and cracks in structures.

The Randomdec signatures for each crack condition were shown on figure 9. At a saw cut depth of 1/8 inch, the Randomdec signature was very similar to the "standard" signature with no crack, although it is not shown. Changes in the signatures initiated and became increasingly obvious when the saw cut became deeper. As observed from figure 9, at the crack depth of 1/4 inch, the change in the signature is sufficiently large to be able to use as a warning device utilizing the voltage of a point on the first peak. This is well ahead of the complete severance of the cross member. Tests have been completed on a 1:14 scale model see figure 10 of an existing offshore platform in the Gulf of Mexico. Fatigue cracks were detected by applying the Random Decrement Technique.

Computer Model

A finite element space frame model of the structure was developed using the GIFTS and the NASTRAN computer programs, see figure 11. The horizontal and the diagonal braces were modeled by beam elements, the top plate was modeled by plate elements which allow bending and membrane flexibility. The total breakthrough of a

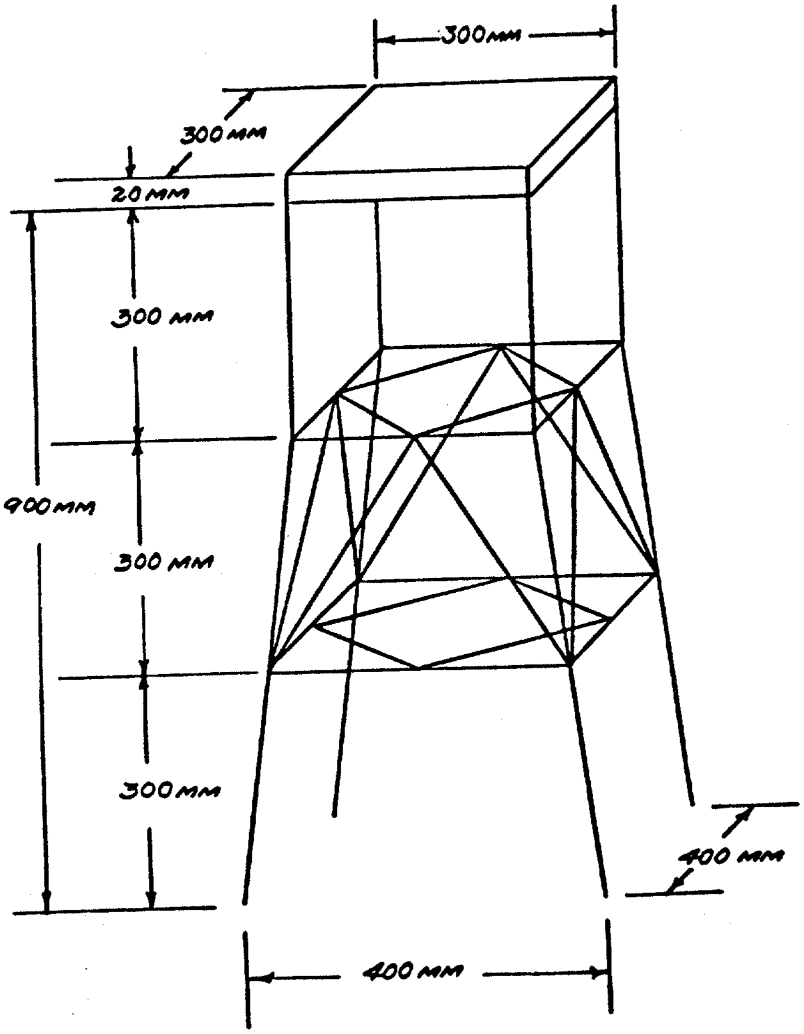


Figure 6: Offshore tower model (dimension)

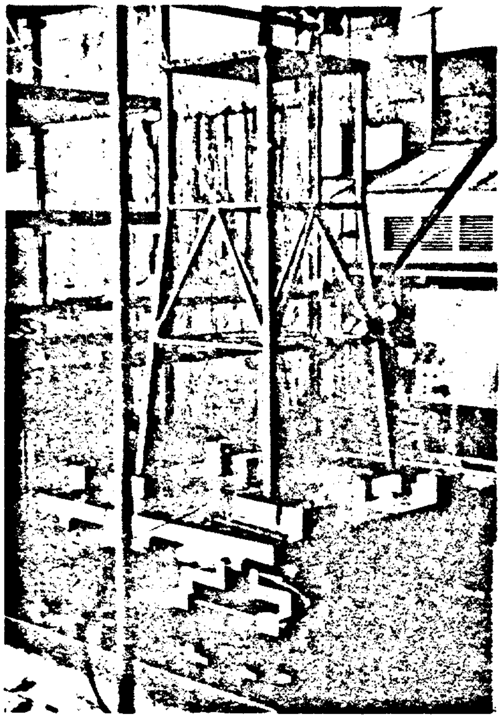


Figure 7: Offshore tower model

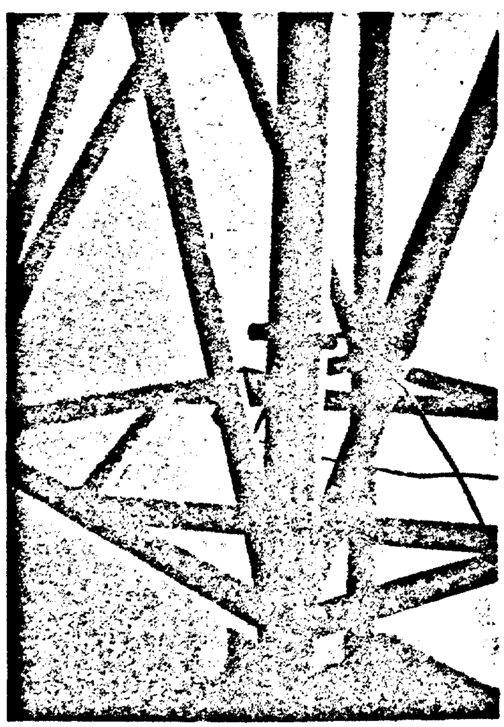


Figure 8: Accelerometer measurement set-up

cross member of the tower model was also reproduced on the finite element computer model. The changes on the dynamic response of the two models were obtained and analyzed.

Another model is being investigated using linear spring elements at the joint in all six directions. Cracks of varying size were simulated by varying the stiffness of the springs.

Computer Analysis Results

Table 1 lists the frequencies of the natural modes of oscillation of the model platform, as they were calculated by the NASTRAN finite element computer program. The first column corresponds to the natural frequencies when there is no defect in the structure, while the second column corresponds to the case where element 28 completely separated at node 9.

One can see that there is a general decrease in the values of the natural frequencies after the separation of one of the branches, and a few additional natural frequencies, which is something to be expected since we now have a less rigid structure.

To get a better understanding of the changes in the dynamic response one has to examine the corresponding eigenvectors (mode shapes) as well. Tables 2 and 3 from the same computer model program, list the eigenvectors corresponding to natural frequency 1 and 2 before the branch separation. It is clear from those two tables that modes #1 and #2 are identical flexural modes of oscillation in the Y and X directions, respectively.

This explains the reason frequency #1 changes from 59.8 Hz to 54.5 Hz after the separation, frequency #2 remains unchanged at 59.8 Hz. Frequency #1 corresponds to a flexural mode in the Y direction, which is the one most significantly affected by the lack of a member branch on the structural frame of plane ZOY.

Tables 4 and 5 list the eigenvectors for modes #1 and #2 after the separation. Node 100 is the new artificial node which was created to model the separation of element 28 at node 9.

From the list of the natural frequencies it is obvious that new modes have been created which confuse the picture and make it very difficult to measure the frequency changes that took place. One could identify groups of identical frequency pairs representing the flexural modes and then search for the same groups in the second column when the member is completely separated. One can then see that the fundamental mode of frequency 59.8 Hz has been shifted downwards by 8.8%, the mode of frequency 109.6 Hz by 1.5%, the mode of frequency 327.0 Hz by 0.09%, the mode of frequency 695. Hz by .57% the mode of frequency 808.6 Hz by 0.59%, the mode of frequency 2,099 Hz by 0.47%, the mode of frequency 4,874 Hz by 0.22%, the mode of frequency 5,440 Hz by .01%, etc. These results then indicate that a complete severance affects mainly the low frequency modes and to a lesser degree the higher frequency modes.

Results from the model with the linear spring elements are being analyzed along with the results from a much finer model with four times the number of elements.

4. CONCLUSIONS

Several features of Randomdec Analysis should be pointed out:

- A very important feature is that the random excitation f(t) does not affect the shape of the Randomdec signature and does not need to be known.
- From this it can be concluded that the natural excitation can be used for the analysis as long as it is reasonably random. This has the advantage that any structure can be tested on-line and does not have to be taken out of service for inspection.
- Randomdec examines the structure in its entire integrity and can therefore detect surface cracks as well as inside defects.
- Randomdec is applicable for both laboratory and field tests. The simple equipment used for the recordings of the response signal make it feasible in many surroundings.

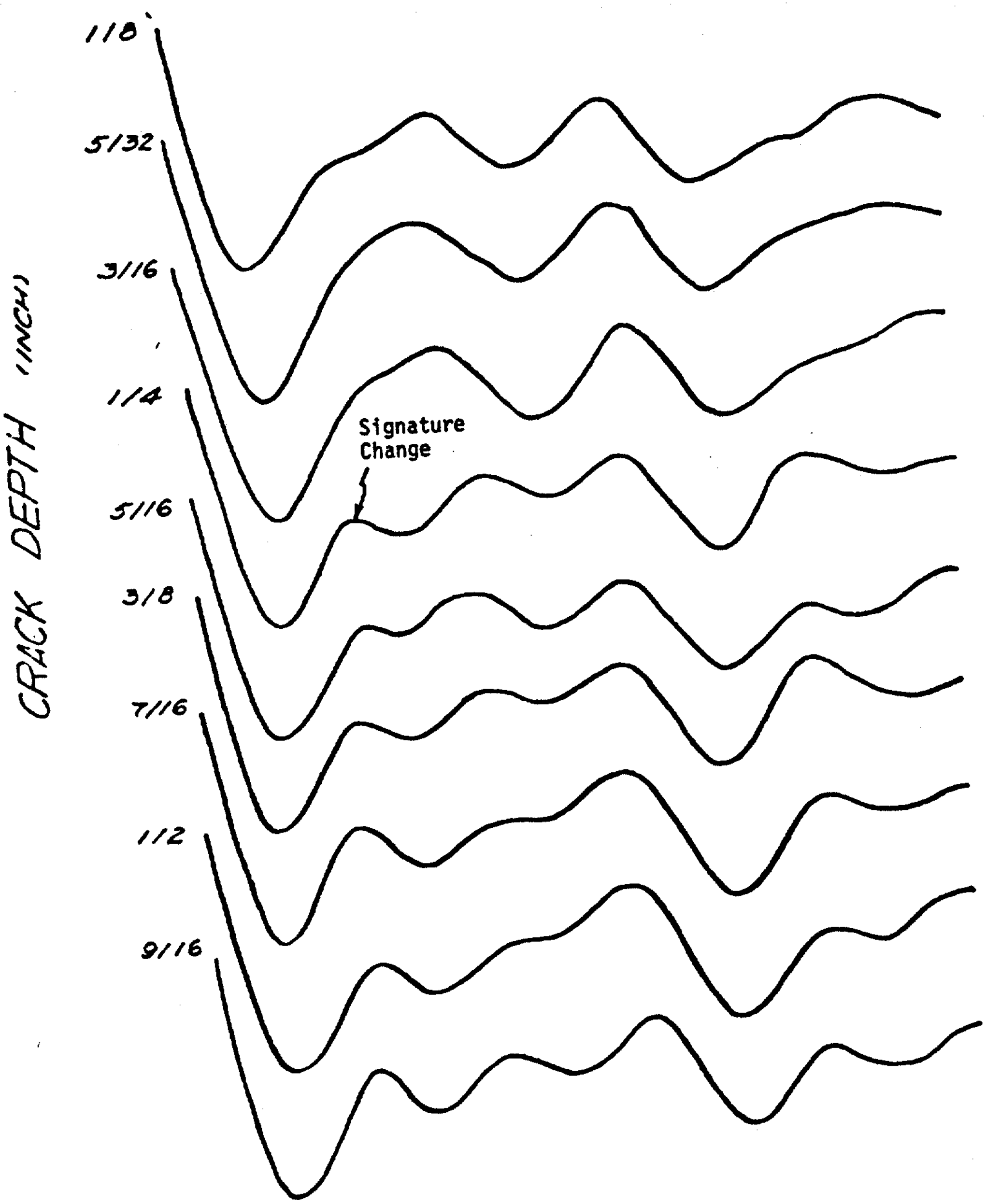


Figure 9: Random decrement signatures

- The recording of response signal takes usually a very short time and can be performed even by untrained people. The main part of the analysis can be done in the laboratory under constant favorable conditions.
- The Randomdec signatures obtained from a fixed crack depth introduced in a structure (model oil platform) is consistent and repeatable. Signatures obtained from each crack depth agreed reasonably well with each other.
- Change in the crack depth as small as 1/16 inch (1.59 mm) in a cross beam of the model structure can be detected with the Randomdec technique.
- The Randomdec signatures demonstrated a marked change in their patterns when the crack depth reached 1/4 inch, which is considerably far ahead from the complete failure of the structure.
- It appeared to be feasible to use the random decrement technique for early detection of incipient flaws and cracks in structures.

5. ACKNOWLEDGEMENTS

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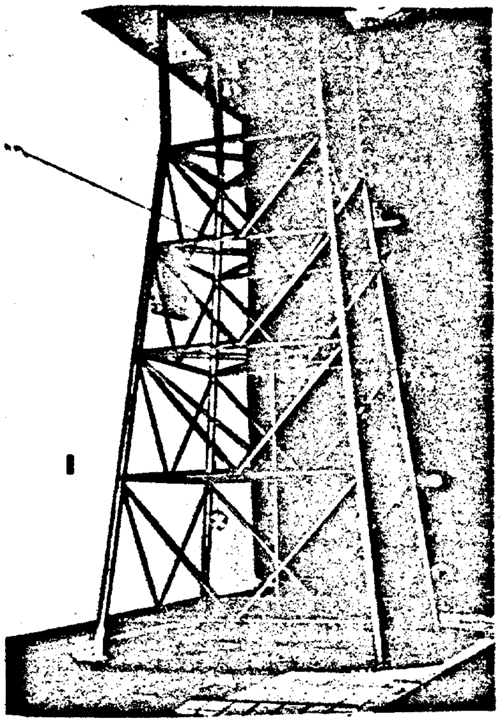

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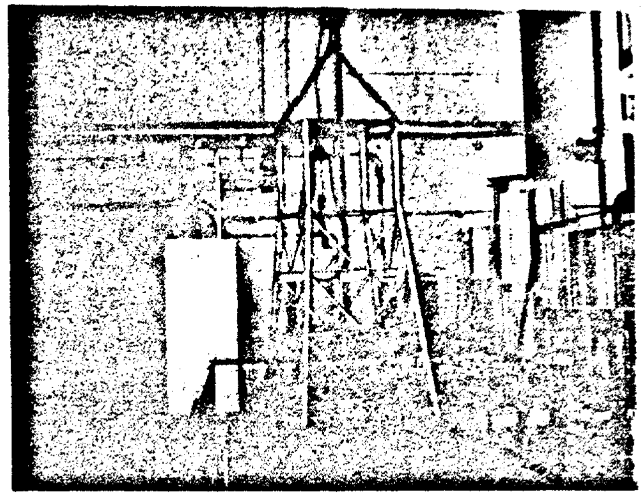

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Figure 10: 1:14 scale offshore platform model

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7. APPENDIX A

It is often convenient to idealize real, continuous systems with multi-degrees of freedom spring-mass-damper systems, which are, in many cases, simpler to treat in a theoretical manner. Nevertheless, they give a deeper insight of the vibrational behavior that is valuable for discrete as well as for continuous systems.

A spring-mass-damper system of m degrees of freedom, subjected to a random input f(t), is under consideration, for which the Randomdec signature of the i^{th} mass has to be extracted. During the k^{th} time segment, the governing equations describing the motion of the system are, using matrix notation,

$$[L]\,\{x(\tau)\}_k = \{f(\tau)\}_k \tag{1}$$

where L is a m by m operator matrix, which is defined by

$$[L] = [M]\,\frac{\delta^2}{\delta\tau^2} + [C]\,\frac{\delta}{\delta\tau} + [K] \tag{2}$$

where M, C and K represent the system mass, damping and stiffness matrices, respectively.

Expanding equation (1) gives, for the i^{th} line:

$$[L_{i1}\, x_1(\tau) + .. + L_{ii}\, x_i(\tau) + .. + L_{im} x_m(\tau)]_k = [f_i(\tau)]_k \tag{3}$$

in order to get the Randomdec signature for the mass i of the system, these time samples have to be averaged with the following initial conditions

$$x_i(0)_k = x_s \qquad k = 1, 2, 3 \ldots.$$
$$\dot{x}_i(0)_k \geq 0 \qquad k = 1, 3, 5 \ldots.$$
$$\dot{x}_i(0)_k \leq 0 \qquad k = 2, 4, 6 \ldots.$$

$$\frac{1}{N}\sum_{k=1}^{N} [L_{i1}\, x_1(\tau) + \ldots + L_{ii}\, x_i(\tau) + \ldots + L_{im}\, x_m(\tau)]_k$$

$$= \sum_{k=1}^{N} [f_i(\tau)]_k \tag{4}$$

The coefficients of the linear operator are independent of time; the summation can therefore be interchanged to give for example

$$\frac{1}{N}\sum_{k=1}^{N} [L_{ij}\, (x_j(\tau))]_k = L_{ij}\, [\frac{1}{N}\sum_{k=1}^{N} x_j(\tau)]_k \tag{5}$$

where the term in brackets on the right hand side of equation (5) is identical to the definition of the

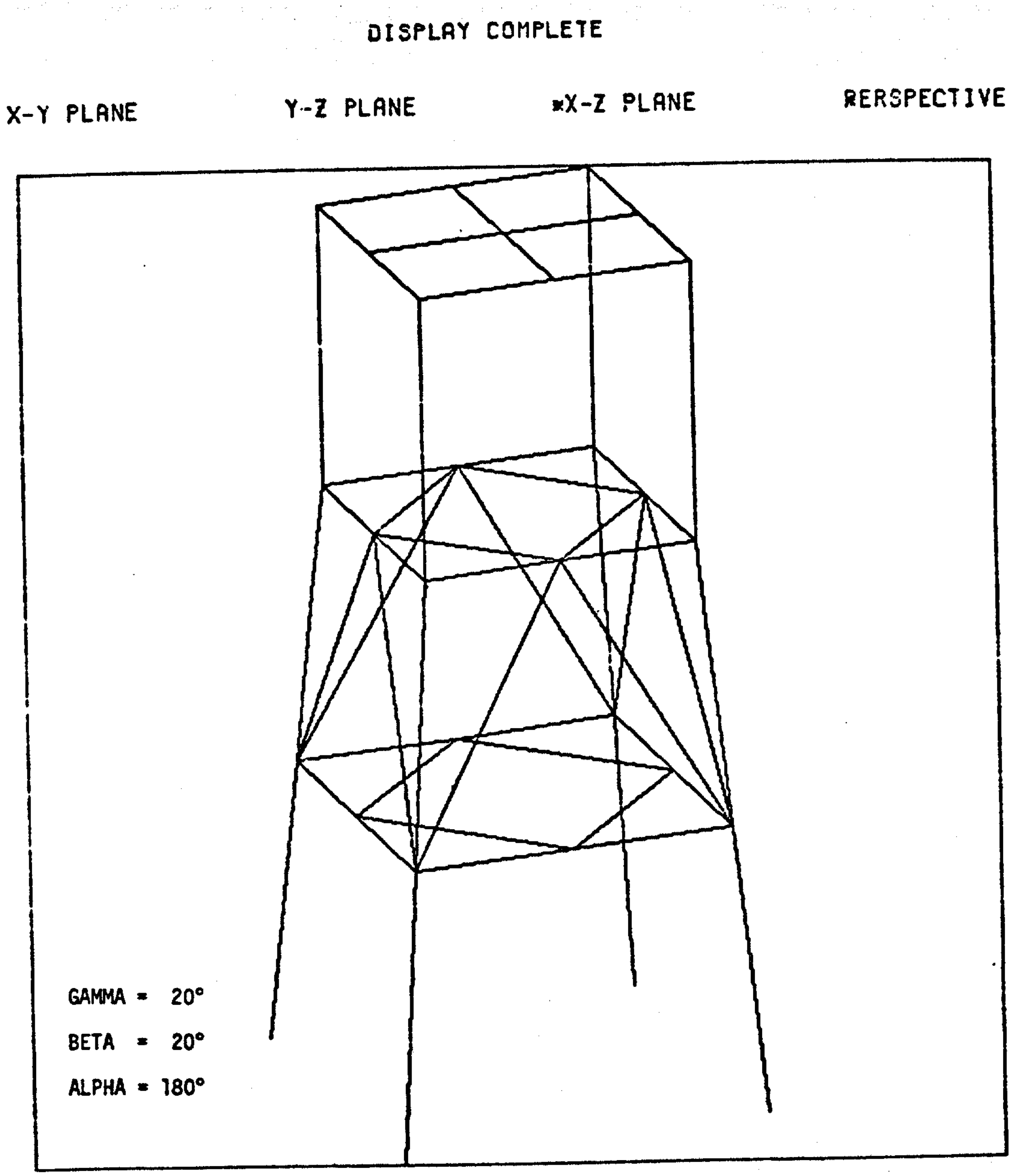

Figure 11: Offshore platform model (NASTRAN)

Randomdec signature δ (τ). Furthermore the right hand sides of equation (4) represent the ensemble averaged random variables, which goes to zero for a large number of N. Equation (4) becomes then, for the i^{th} row,

$$L_{i1}\,[\delta_1] + L_{12}\,[\delta_2] + \ldots + L_{im}[\delta_m] = 0 \qquad (6)$$

or simply, in matrix form,

$$[L]\,\{\delta\} = 0 \qquad (7)$$

which is identical to the equations governing the free vibration of the system. In order to define unique solutions, initial conditions have to be set up for each mass m_j, $j \neq i$. For the i^{th} mass, they are

$$\delta_i(0) = x_s$$
$$\dot{\delta}_i(0) = 0$$

The remaining initial conditions for the other masses can be determined by an energy approach. In a holonomic system lagrange's equation can be applied (6), giving

$$\frac{\delta}{\delta t}\left(\frac{\delta T}{\delta \dot{x}_i}\right) - \frac{\delta T}{\delta x_i} + \frac{\delta V}{\delta x_i} = Q_{nc} \qquad (8)$$

where T and V are the kinetic and potential energy expressions, while Q_{nc} denomincates the nonconservative forces of the system. Furthermore the following relations are valid for a spring mass damper system:

$$T = T(\dot{x}_1^2,\ \dot{x}_2^2,\ \ldots\ \dot{x}_m^2) \qquad (9)$$

$$\frac{\delta T}{\delta \dot{x}_i} = \frac{\delta T}{\delta \dot{x}_i}\,(\dot{x}_1,\ \dot{x}_2,\ \ldots\ \dot{x}_m) \qquad (10)$$

$$\frac{\delta T}{\delta x_i} = 0 \tag{11}$$

$$V = V(x_1^2,\ x_2^2,\ \dots\ x_m^2,\ x_1 x_2,\ \dots\ x_i x_j) \tag{12}$$

$$\frac{\delta V}{\delta x_i} = \frac{\delta V}{\delta x_i}\ (x_1,\ x_2,\ \dots\ x_m) \tag{13}$$

The term on the right hand side of equation (8), the non-conservative forces, come from the damping alone and are, by definition, proportional to the velocities:

$$Q_{nc} = Q_{nc}(\dot{x}_1,\ \dot{x}_2,\ \dots\ \dot{x}_m) \tag{14}$$

For $\tau = 0$ the average initial velocities $\dot{\delta}_j(0)$ of the remaining masses can be taken to zero, since their mean value is zero and they are sampled at $\tau = 0$ where $\dot{x}_i(0)$ is both positive and negative. This can be included in the lagrange equation (8), where, for $\tau = 0$, all terms drop out except the potential energy expression:

$$\left(\frac{\delta V}{\delta x_i}\right)\Bigg|_{\tau=0} = 0 \tag{15}$$

which means that, for $\tau = 0$, V is minimal and therefore all the masses m_j are in a position of static equilibrium. The signature, therefore, has the same shape as the free vibration response of the system due to an initial displacement of the mass m_i and all the other masses m_j in their respective equilibrium position.

Table 1

Real Eigenvalues

Mode No.	Uncracked Cycles/s	Cracked Cycles/s
1	5.985427+01	5.455365+01
2	5.985434+01	5.982981+01
3	7.378253+01	7.178627+01
4	1.096753+02	1.079195+02
5	1.096753+02	1.095503+02
6	1.401590+02	1.378930+02
7	2.656457+02	2.374263+02
8	3.020945+02	2.748728+02
9	3.205365+02	2.966485+02
10	3.270113+02	3.031506+02
11	3.270117+02	3.032749+02
12	3.414802+02	3.269990+02
13	4.644798+02	3.273253+02
14	6.476650+02	3.363414+02
15	6.476650+02	3.422305+02
16	6.950135+02	4.645995+02
17	6.950135+02	6.496490+02
18	6.958005+02	6.557123+02
19	7.797218+02	6.910069+02
20	7.813911+02	6.955249+02
21	7.919391+02	6.996401+02
22	8.088028+02	7.733953+02
23	8.088033+02	7.872478+02
24	8.341894+02	7.915182+02
25	8.370232+02	8.038068+02
26	8.370232+02	8.086494+02
27	8.817422+02	8.350940+02
28	9.642078+02	8.355844+02
29	1.036857+03	8.381134+02
30	1.109549+03	8.849877+02
31	1.109549+03	9.870237+02
32	1.142433+03	1.044649+03
33	1.154169+03	1.110238+03
34	1.167893+03	1.131118+03
35	1.297294+03	1.152613+03
36	1.297294+03	1.160173+03
37	1.337202+03	1.275100+03
38	1.440242+03	1.301650+03
39	1.440242+03	1.337754+03
40	1.507570+03	1.401835+03
41	1.647827+03	1.442093+03
42	1.649951+03	1.505987+03
43	1.649951+03	1.638476+03
44	1.686638+03	1.648700+03
45	1.693084+03	1.650189+03
46	2.075295+03	1.670505+03
47	2.099964+03	1.692517+03
48	2.099964+03	1.975913+03
49	2.226976+03	2.078267+03
50	2.286140+03	2.099548+03
51	2.363273+03	2.109878+03
52	2.363273+03	2.226212+03
53	2.606871+03	2.327376+03
54	2.606871+03	2.363253+03
55	2.989672+03	2.363656+03
56	3.222953+03	2.619098+03
57	3.282000+03	2.654583+03
58	3.282000+03	2.988340+03
59	3.350284+03	3.227629+03
60	3.557201+03	3.286070+03
61	3.566257+03	3.291030+03
62	3.566257+03	3.361859+03
63	3.584918+03	3.561822+03
64	3.587878+03	3.577364+03
65	4.605922+03	3.584951+03
66	4.858828+03	3.728487+03
67	4.874586+03	4.374977+03
68	4.874586+03	4.607465+03
69	4.899032+03	4.863201+03
70	4.964329+03	4.874673+03
71	5.259325+03	4.890971+03
72	5.259325+03	4.991822+03
73	5.348680+03	5.094332+03
74	5.348680+03	5.303557+03
75	5.440220+03	5.344952+03
76	5.440220+03	5.352518+03
77	5.486620+03	5.439886+03
78	5.725430+03	5.440242+03
79	6.046908+03	5.465077+03
80	6.109739+03	5.725145+03
81	8.091969+03	6.046908+03
82	8.358225+03	6.109734+03
83	8.358225+03	8.091968+03
84	8.358356+03	8.241454+03
85	8.358811+03	8.358280+03
86	8.386183+03	8.358463+03
87	8.386183+03	8.359567+03
88	8.765246+03	8.360433+03
89	1.064645+04	8.386185+03
90	1.089508+04	8.386189+03
91	1.094631+04	8.765246+03
92	1.094631+04	1.064645+04
93	1.179240+04	1.089508+04
94	1.179336+04	1.094631+04
95	1.179336+04	1.094631+04
96	1.179412+04	1.179266+04
97	1.211822+04	1.179334+04
98	1.286380+04	1.179389+04
99	1.286380+04	1.211822+04
100		1.249050+04
101		1.286380+04
102		1.286380+04

Table 2

Uncracked Model

Real Eigenvector No. 1

Eigenvalue = 59.8 Cycles/Sec

Point ID.	T1(X)	T2(Y)	T3(Z)
1	6.389189-01	-9.997100-01	1.222257-02
2	6.390413-01	-9.997883-01	5.551883-02
3	6.389189-01	-9.997100-01	-1.222257-02
4	6.390413-01	-9.997883-01	-5.551883-02
5	4.475486-01	-7.001066-01	1.265031-02
6	4.472708-01	-6.999291-01	5.746180-02
7	4.475486-01	-7.001066-01	-1.265031-02
8	4.472708-01	-6.999291-01	-5.746180-02
9	3.550190-01	-5.552163-01	1.286317-02
10	3.518110-01	-5.509728-01	3.742287-02
11	3.544854-01	-5.548752-01	5.842865-02
12	3.521490-01	-5.504440-01	2.391847-02
13	3.550190-01	-5.552163-01	-1.286317-02
14	3.518110-01	-5.509728-01	-3.742287-02
15	3.544854-01	-5.548752-01	-5.842865-02
16	3.521490-01	-5.504440-01	-2.391847-02
17	3.539709-01	-5.530252-01	1.299645-02
18	3.522461-01	-5.519229-01	5.903404-02
19	3.539709-01	-5.530252-01	-1.299645-02
20	3.522461-01	-5.519229-01	-5.903404-02
21	3.652303-01	-5.713373-01	1.404212-02
22	3.652555-01	-5.717626-01	3.958115-02
23	3.650087-01	-5.711956-01	6.378381-02
24	3.654366-01	-5.714793-01	2.529791-02
25	3.652303-01	-5.713373-01	-1.404212-02
26	3.652555-01	-5.717628-01	-3.958115-02
27	3.650087-01	-5.711958-01	-6.378381-02
28	3.654366-01	-5.714793-01	-2.529791-02
29	.0	.0	.0
30	.0	.0	.0
31	.0	.0	.0
32	.0	.0	.0
33	6.391255-01	-9.998543-01	-2.076362-02
34	6.390473-01	-9.999766-01	3.248679-02
35	6.391255-01	-9.998543-01	2.076362-02
36	6.390473-01	-9.999766-01	-3.248679-02
37	6.391404-01	-1.000000+00	-1.551409-17

Table 3

Uncracked Model

Real Eigenvector No. 2

Eigenvalue = 59.8

Point ID.	T1(X)	T2(Y)	T3(Z)
1	9.997883-01	6.390413-01	-5.551883-02
2	9.997100-01	6.389189-01	1.222257-02
3	9.997883-01	6.390413-01	5.551883-02
4	9.997100-01	6.389189-01	-1.222257-02
5	6.999291-01	4.472708-01	-5.746180-02
6	7.001066-01	4.475488-01	1.265031-02
7	6.999291-01	4.472708-01	5.746180-02
8	7.001066-01	4.475488-01	-1.265031-02
9	5.548752-01	3.544854-01	-5.842865-02
10	5.504440-01	3.521490-01	-2.391847-02
11	5.552163-01	3.550190-01	1.286317-02
12	5.509728-01	3.518110-01	3.742287-02
13	5.548752-01	3.544854-01	5.842865-02
14	5.504440-01	3.521490-01	2.391847-02
15	5.552163-01	3.550190-01	-1.286317-02
16	5.509728-01	3.518110-01	-3.742287-02
17	5.519229-01	3.522461-01	-5.903404-02
18	5.530252-01	3.539709-01	1.299645-02
19	5.519229-01	3.522461-01	5.903404-02
20	5.530252-01	3.539709-01	-1.299645-02
21	5.711956-01	3.650087-01	-6.378381-02
22	5.714793-01	3.654366-01	-2.529791-02
23	5.713373-01	3.652303-01	1.404212-02
24	5.717626-01	3.652555-01	3.958115-02
25	5.711956-01	3.650087-01	6.378381-02
26	5.714793-01	3.654366-01	2.529791-02
27	5.713373-01	3.652303-01	-1.404212-02
28	5.717626-01	3.652555-01	-3.958115-02
29	.0	.0	.0
30	.0	.0	.0
31	.0	.0	.0
32	.0	.0	.0
33	9.999766-01	6.390473-01	-3.248679-02
34	9.998543-01	6.391255-01	-2.076362-02
35	9.999766-01	6.390473-01	3.248679-02
36	9.998543-01	6.391255-01	2.076362-02
37	1.000000+0	6.391404-01	7.873543-17

Table 4

Cracked Model

Real Eigenvector No. 1

Eigenvalue = 54.5 Cycles/Sec

Point ID.	T1(X)	T2(Y)	T3(Z)
1	2.256729-01	-1.000000+00	5.406098-02
2	2.257286-01	-6.702547-01	2.499385-02
3	-1.039019-01	-6.702576-01	-2.216022-02
4	-1.037968-01	-9.998061-01	-2.574997-02
5	1.286834-01	-9.200148-01	5.482133-02
6	1.236920-01	-4.491532-01	2.598915-02
7	-5.370587-02	-4.485168-01	-2.304104-02
8	-5.382929-02	-6.270659-01	-2.664056-02
9	7.775391-02	-8.153091-01	5.519783-02
10	7.669615-02	-4.264953-01	3.000532-02
11	7.726533-02	-3.402974-01	2.648506-02
12	-1.157327-02	-3.374111-01	-1.222330-03
13	-2.856505-02	-3.403585-01	-2.347991-02
14	-2.812149-02	-3.537061-01	-2.384857-02
15	-2.871493-02	-4.445427-01	-2.708407-02
16	6.016503-02	-4.399361-01	-5.829691-03
17	7.451042-02	-7.202766-01	4.796040-02
18	7.015273-02	-3.304531-01	2.564225-02
19	-2.342802-02	-3.317048-01	-2.282405-02
20	-2.588566-02	-4.242756-01	-2.613719-02
21	8.195456-02	-4.664983-01	2.691140-02
22	8.199097-02	-4.036171-01	2.674195-02
23	8.153223-02	-3.484576-01	2.922586-02
24	2.794881-02	-3.488713-01	3.132349-03
25	-3.171204-02	-3.486015-01	-2.590492-02
26	-3.218699-02	-4.099303-01	-2.697526-02
27	-3.200155-02	-4.632268-01	-2.988059-02
28	2.162405-02	-4.646502-01	3.410681-02
29	.0	.0	.0
30	.0	.0	.0
31	.0	.0	.0
32	.0	.0	.0
33	6.091636-02	-9.999728-01	-5.173192-03
34	2.257161-01	-8.352498-01	3.979212-02
35	6.090577-02	-6.703316-01	-5.897067-04
36	-1.038692-01	-8.352228-01	-2.209736-02
	6.090467-02	-8.352522-01	2.098019-03
100	1.106762-01	-4.399579-01	7.847411-02

Table 5

Cracked Model

Real Eigenvector No. 2

Eigenvalue = 59.8

Point ID.	T1(X)	T2(Y)	T3(Z)
1	9.999002-01	6.813270-02	-3.610747-02
2	9.998901-01	6.999780-02	3.146085-02
3	9.979109-01	7.012020-02	3.609367-02
4	9.979021-01	6.801050-02	-3.144064-02
5	7.001428-01	4.788021-02	-3.737256-02
6	6.992555-01	4.876294-02	3.256191-02
7	6.985066-01	4.848744-02	3.736192-02
8	6.983632-01	4.801909-02	-3.254495-02
9	5.536276-01	3.798001-02	-3.800209-02
10	5.491473-01	3.649809-02	-2.460452-03
11	5.537673-01	3.847473-02	3.310981-02
12	5.482484-01	3.789892-02	3.722437-02
13	5.536470-01	3.795860-02	3.799304-02
14	5.490798-01	3.933625-02	2.716887-03
15	5.536691-01	3.835653-02	-3.309448-02
16	5.512085-01	3.776422-02	-3.750900-02
17	5.501215-01	3.684260-02	-3.829341-02
18	5.509838-01	3.878721-02	3.344017-02
19	5.509687-01	3.699378-02	3.836787-02
20	5.512146-01	3.879627-02	-3.344470-02
21	5.698690-01	3.906043-02	-4.148242-02
22	5.700895-01	3.937784-02	-2.753983-03
23	5.698854-01	3.939135-02	3.614390-02
24	5.704374-01	3.929008-02	3.945165-02
25	5.697959-01	3.916164-02	4.147681-02
26	5.700188-01	3.916578-02	2.669998-03
27	5.698263-01	3.930689-02	-3.613831-02
28	5.702253-01	3.920364-02	-3.938356-02
29	.0	.0	.0
30	.0	.0	.0
31	.0	.0	.0
32	.0	.0	.0
33	9.991280-01	6.807875-02	-3.238372-02
34	1.000000+00	6.908123-02	-2.129885-03
35	9.991278-01	7.006651-02	3.239193-02
36	9.980115-01	6.908128-02	2.223126-03
37	9.991513-01	6.908294-02	2.994066-05
100	6.787433-01	3.776647-02	-4.650651-02