

## A CONSTITUTIVE EQUATION FOR NONLINEAR STRESS-STRAIN CURVES IN ROCKS AND ITS APPLICATION TO STRESS ANALYSIS AROUND A BOREHOLE DURING DRILLING

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### ABSTRACT

The stress-strain behavior of Berea, Ohio, and Pecos sandstone was measured intensively under triaxial loading paths. Both compression and extension tests were performed at different pore fluid pressures. These tests showed that four modes can arise in the stress-strain curves up to failure: an initial nonlinear portion; linear portion; a final nonlinear portion; and volume change of the rock matrix due to pore fluid pressure. The characteristics of these modes were studied, and a nonlinear model was developed to represent the important characteristics, i.e., the four modes, of the behavior of sandstones by means of a constitutive equation in such a way as to satisfy certain mathematical requirements.

A finite element model using nonlinear stress-strain relationship was developed which simulates the stress state of a structure up to failure. The finite element model was used to predict fracture pressure around a borehole during drilling. The shape of the borehole is either circular or elliptic, and the tectonic stress is directional. Since the nonlinear stress state depends upon loading history, the loads were applied incrementally, simulating phenomena occurring during the drilling process. It is shown that use of conventional linear stress-strain behavior gives significantly erroneous results for well stability problems.

### INTRODUCTION

The finite element method has now become recognized as a general method of wide applicability to engineering and physical science problems. As a result of its broad applicability and systematic generality, the method has gained wide acceptance by civil engineers and architectural engineers for designing rock structures and soil foundations.

Experimental studies<sup>7</sup> on rock and soil behavior have been conducted since the early 20th century. Numerous data of the effect of temperature, pore pressure, stress state and time upon rock and soil behavior have been published. In the 1960's, various simple nonlinear constitutive equations<sup>8,9</sup> were proposed. Although these were helpful for analytical use, the development of sophisticated numerical discretization techniques need not require simplified constitutive equations.

Most nonlinear equations were developed in the past to fit nonlinear stress-strains for metals and soils. Consequently, one often encounters trouble applying them to porous materials consisting of grains. Rock exhibits volumetric expansion and compression and also increased rigidity as hydrostatic pressure increases. For instance, the nonlinearity of sandstones is significant in radial strain rather than axial strain for triaxial compression tests. This is due to volumetric expansion caused by the sliding between grains before failure of rock occurs. Since the rigidity increases with hydrostatic pressure, the usual deviatoric stress-strain relations can not be applied to sandstones. Various nonlinear constitutive stress-strain relationships were tested by the present authors before this work was initiated. These include equations proposed by Barla,<sup>1</sup> Drucker,<sup>5,6</sup> Von Mises<sup>8</sup>, Prandtl<sup>9</sup>, Mroz<sup>12</sup> and Dienes<sup>4</sup>. But none of them could simultaneously take into account nonlinear volumetric change and increased rigidity due to hydrostatic pressure for both compression and extension. Although the nonlinear equations proposed by Sandler<sup>3</sup> and Prevost<sup>11</sup> partially satisfy these requirements, they do not represent accurately both axial and radial stress-strain of sandstones with pore pressure under triaxial extension and compression loading. In this work, the nonlinear constitutive equation was assumed to be composed of four parts: an initial nonlinear part, an elastic part, a plastic part, and volume change of the rock matrix due to fluid pressure. Each part was carefully examined and expressed by mathematically consistent equations. Thus, a nonlinear constitutive equation was established which closely simulates the nonlinear behavior of sandstones with pore pressure and was applied to a finite element simulator.

Estimation of fracture gradient is important for the drilling operation since it limits the maximum density of the drilling fluid. It also gives the maximum borehole strength at the casing shoe during kick control operations. Estimation of the fracture gradient is erroneous if one simply applies the linear stress-strain behavior. The stress state changes on a wide range before borehole rupture, including significant nonlinearity in stress-strain relations for the rock. In this work, the fracture gradient around a wellbore was studied, as an example of the practical application of the finite element model using nonlinear stress-strain. The minimum fracture gradient usually occurs when the surface of the wellbore cracked or notched. In this case the fracture gradient is close to the tectonic stress gradient. On the other hand,

References and illustrations at end of paper.