# DYNAMIC RESPONSE OF OFFSHORE STRUCTURES

Annual Technical Report
Research Supported by the Minerals Management Service
Technology Assessment Program

Under USGS Contract No. 14-08-0001-20665
Period of Contract and Report:
September 1982 to August 1983
Contract Amount $64,000

Prof. J. Kim Vandiver
Massachusetts Institute of Technology
Room 5-222
Cambridge, Massachusetts 02139

November 1, 1983

The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the U.S. Government.

## INTRODUCTION

The research described in this report deals with the dynamic response prediction of offshore structures. The project had two specific tasks: 1) the development of a new technique for the simulation of random wave forces in conjunction with finite element computer models of offshore structures. 2) The development of a theoretical model for the prediction of hydrodynamic viscous damping of an offshore structure in the presence of both waves and current. The results of both topics are completely described in graduate student theses which have been recently completed. The wave force simulation research is the subject of a doctoral dissertation (1) and the damping research was the subject of a master's thesis (2). The principal results will be discussed here. Interested parties may request copies of the theses themselves.

## ACKNOWLEDGEMENTS

The author would like to thank Mr. John Gregory and Mr. Charles Smith of the Minerals Management Service for their interest in and continued commitment to research in deepwater offshore engineering. Such research promotes safe development of mineral resources on the outer continental shelves.

## THE SIMULATION OF WAVE FORCES

Current industry practice in the modelling of dynamic response of offshore structures makes extensive use of finite element techniques. The wave forces are generally computed using the Morison equation. This requires that water particle velocities and accelerations be known over an extensie grid of points corresponding to structural elements. The forces on each structural component are computed and applied at the appropriate nodes of the finite element model. To properly account for non-linear mechanisms such as due to drag forces and the actual position of the free structure, these computations must be done in the time domain.

The simulation of a unique event, such as the design wave is usually accomplished by prescribing a deterministic non-linear wave or series of waves. Present practice seems quite adequate as the duration of the event is short and not too much computer time is required. For the purpose of fatigue life estimation, long time histories of random waves must be generated. To obtain reliable statistics of, for example stress at a critical node, requires more computer time than is generally budgeted. The principal source of numerical inefficiency with current technology is in the wave force computation.

By present practice the wave induced water particle velocities and accelerations are generated by summing a finite number of sinusoidal time histories with random initial phase angles. The amplitude of each sine wave is selected so that the sum provides an approximation to a desired wave spectrum. The more components used the better the approximation. A typical simulation may use 256 components. An excessive amount of computer time is required. Another disadvantage of this approach is that because each component itself is deterministic, the sum eventually repeats itself.

In this research a method has been developed which simulates wave induced velocities and accelerations in a numerically efficient non-repeating way. Furthermore, the desired wave spectrum is approximated, not at a finite number of discrete frequencies but by a continuous wave spectrum. This is beneficial in the simulation of slowly varying drift forces which depend on difference frequencies between various components of the wave spectrum. Finite numbers of components yield drift forces at only a finite number of frequencies, not a continuum as realized in the real ocean. The new method provides a continuum of sum and difference frequencies.

The generation of water particle velocities and accelerations at a single point is not adequate. The water particle kinematics must be known simultaneously over a large grid of points corresponding to a complete three dimensional structure. This is a straightforward though inefficient computation when the discrete sinusoidal components are known, because the attenuation of water particle velocities and accelerations with depth may be computed for each component individually. Likewise, the phase shift due to the horizontal propagation of the waves may be computed for each individual component before summing the components to obtain the total kinematics.

In the method developed in this research, the random wave time series is generated at an origin or source point on the surface. The basic technique is simply the passage of band limited white noise through a digital filter. The output of the filter is a random wave time history with the desired spectral shape. Digital filter techniques have been available for years. It is reasonable to question why they haven't replaced the technique of summing sinusoidal components long ago. The answer is that because the individual components are not known, it has not been possible to propagate the waves to other locations separated horizontally and vertically from the source point. One of the principal contributions of this research has been in the development of numerical techniques which properly account for dispersive horizontal propagation of random waves as well as the exponential attenuation of waves with increasing depth.

In this research, a vertical velocity time history with the desired spectral shape is generated at a point defined as the origin located on the mean waterline. A particular type of digital filter known as an auto-regressive moving average (ARMA) filter is used. The output of the ARMA filter is processed by a series of numerical convolutions. Each convolution accounts for a vertical or horizontal shift to a different spatial location. Acceleration at each point is obtained by a numerical differentiation of the vertical velocity. Horizontal velocities and accelerations are obtained by use of a Hilbert transform which is also implemented as a numerical convolution. These steps are shown schematically in Figure 1, and discussed individually below.

**ARMA Simulation of Vertical Wave Particle Velocity**

The ARMA technique is a linear digital filter in which weighted values of the past N realizations of the velocity are summed with the current and past M values of the white noise input as shown in Equation 1. The coefficients $a_n$ are the weighting coefficients for the past values of the output velocity V(t-nDt). The $b_m$ coefficients are the weighting values of the past and present values of the input noise, W(t-mDt), where Dt is the time step chosen. The $a_n$ and $b_m$ are chosen so as to give the transfer function of the filter the shape of the desired wave velocity spectrum.

$$V(t) = \sum_{n=1}^{N} a_n\, V(t-nDt) + \sum_{m=0}^{M} b_m W(t-mDt)$$

The coefficients, $a_n$ and $b_m$ are difficult to obtain for each specified wave spectrum. Once known, however, they need not be computed again, whenever that spectral shape is desired. In this research, the ARMA coefficients representing a family of Bretschneider spectra have been found. Figure 2 is a plot of the target Bretschneider spectrum and the transfer function of the corresponding ARMA filter. The two curves are nearly indistinguishable. A task which remains to be done is a solution for the ARMA coefficients for other popular wave spectra such as the JONSWAP.

## Vertical Attenuation of Wave Kinematics

From the mean free surface, deepwater waves are attenuated exponentially at a rate which is determined by wave number. In the frequency domain the transfer function is as given in Equation 2.

$$G(f,z) = e^{-(2\pi f)^2 z/g} \qquad 2$$

where

f = wave frequency in Hz

g = acceleration of gravity

z = distance measured positive downward from the mean free surface.

K = $(2\pi f)^2/g$ , the wave number

This transfer function introduces no phase shift. The Fourier transform of G(f,z) is known and is given in Equation 3.

$$g(t,z) = \sqrt{g/2\pi z}\; e^{-(2t^2/4z)} \qquad 3$$

This is the impulse response function corresponding to G(f,z). When sampled at a finite number of points this function may be implemented in a discrete form as given in Equation 4, and used in a numerical convolution to obtain the properly attenuated wave velocity at any depth Z.

$$g_d(z,kDt) = \sqrt{\frac{gDt^2}{4\pi z}}\ \exp\left(-\frac{gDt^2k^2}{4z}\right) \qquad 4$$

The convolution sum is as shown in Equation 5.

$$V(t,z) = V(t,0) * g(t,z)$$

$$V(z,nDt) = \sum_k V(0,(n-k)Dt)g_d(kDt,z) \qquad 5$$

To demonstrate the accuracy of this convolution a time series representing the vertical velocity on the mean free surface was generated using a sum of 50 sinusoidal components. A sample time history is shown in Figure 3. Using the convolution, the corresponding time history 20 meters down was computed and compared to the result obtained by summing the individually attenuated sinusoidal components. The result is shown in Figure 4. The two time histories are indistinguishable.

**Horizontal Wave Propagation**

The horizontal propagation of a random wave train must properly account for dispersion; that is the fact that waves of different frequency travel at different speeds. The transfer function which accounts for this must introduce no wave attenuation with distance travelled, but must account for phase shift. The proper frequency domain expression of this transfer function is given in Equation 6.

$$H(f,x) = e^{-j\frac{(2\pi f)^2 x}{g} \cdot \mathrm{sign}(f)} \qquad 6$$

where sign(f) means the sign of f.

The inverse Fourier transform of this expression is not known analytically and must be obtained for each case numerically. If h(x,kDt) is the result of the discrete inverse Fourier transform then the horizontal propagation with dispersion may be given as the following convolution.

$$V(nDt,x,z) = \sum_{k=-N}^{N} V((n-k)Dt,0,z)h(x,kDt) \qquad 7$$

Figure 5 is an example of h(x,kDt) for a horizontal propagation of 100m. Figure 6 is a comparison of the numerically exact propagation of 50 sinusoidal components with the numerical convolution. The horizontal distance is 100m which is approximately 15 wavelengths of the shortest wave in the summation. The convolution is designed so as to not allow the phase error at the shortest wave to exceed $\pi/4$.

## Hilbert Transform and Differentiation

Once the vertical velocity at a remote point (x,z) is known, it is still necessary to obtain the horizontal water particle velocity and the vertical and horizontal components of acceleration. The horizontal velocity is obtained by a $90^{\circ}$ phase shift from the vertical. This is done by use of the Hilbert transform in the time domain by means of yet another numerical convolution. Both components of acceleration are then obtained by a simple central difference differentiation carried out in the time domain.

## Numerical Efficiency

Comparisons of the new methods described here with the sum of sinusoids have been conducted for two cases. In the first case a 10 row by 10 column grid of one hundred (x,z) points is considered. In the second a 100 by 100 grid of 10,000 points is evaluated. For each use the number of arithmetical calculations, the number of necessary memory storage locations, and the number of memory input-output transfers are estimated.

The new methods are compared to the sum of sinusoids for the case of 256 components. The results are presented in Table 1 in terms of the ratio of the number required for the sinusoidal method compared to the new method.

TABLE 1

RATIO OF REQUIREMENTS FOR SUM OF SINUSOIDS COMPARED TO NUMERICAL CONVOLUTION

| | Grid 10 x 10 | Spacing 100 x 100 |
|---|---|---|
| Arithmetical Calculations | 24 | 31 |
| Memory Storage Locations | 33 | 1484 |
| Input/Output Transfers | 256 | 256 |

Depending on the item considered the convolution method is 10 to 1000 times more efficient. No actual run time comparisons have been made and no effort has as yet been made to optimize the numerical methods used in the convolution approach. The savings are potentially so significant that further development of the technique are fully justified.

**Future Research**

These methods may be improved and extended in several ways. First ARMA coefficients necessary for modelling most of the popular wave spectra should be evaluated. Second, optimization of some of the convolution algorithms should be undertaken. The techniques should be extended to account for shallow water effects and the modelling of directionally spread random seas. Methods for modelling wave forces up to the instantaneous position of the free surface should be investigated. A simple application of a stretched linear model has been tried and appears to work.

The new methods can be used to evaluate the wave kinematics on a structure using measured wave data as an input. In actual field tests the waves must be measured at a distance from the structure. For the first time these techniques give us the ability to calculate the wave kinematics at the structure using the remote measurement as an input. For the prediction of response of floating structures such as TLP's, it will be necessary to develop methods for evaluating hydrodynamic pressure as well as velocities and accelerations.

## Hydrodynamic Viscous Damping in Waves and Current

In previous research sponsored by the USGS it was shown that the damping of a vibration mode of an offshore structure was dependent on seastate. In that research the technique of Gaussian closure was used to solve the non-linear equations of motion for a structure excited by waves. The non-linearity resulted from the drag force term of the Morison wave force equation. The results of that research showed that the damping of, for example, the fundamental bending mode of an offshore platform depended on the root mean square relative water particle velocity by the structure. In other words, the energy lost to the generation of turbulence in the water, depended on seastate. The higher the seastate the higher the damping. The results of the research were confirmed in a model test. Those results were valid only for the case of no current. Unfortunately, for many of the newer design concepts the influence of current must be taken into consideration. The long period surge and sway motions of guyed towers, TLP's and semi-submersibles have the effect of an apparent quasi-steady current when one is concerned with the response at higher frequencies, such as the first bending mode of a guyed tower. In addition, high seas are always associated with surface currents.

In this research, the method of stochastic linearization was applied to the estimation of viscous hydrodynamic damping in waves and current. The theoretical results are demonstrated here in example calculations of the damping of the first bending mode of a single pile structure and for the first bending mode of a guyed tower.

Figure 7 is a drawing of an existing Gulf of Mexico structure. The mode shape of the first bending mode is indicated. Figure 8 is a plot of the predicted viscous hydrodynamic damping ratio $\xi_v$ for the first bending mode as a function of significant wave height $H_{1/3}$ and current speed, V.

Figure 9 is a diagram of a guyed tower, with drawings of both the sway and first bending modes. The natural periods in these modes are 31 seconds in sway and 4.5 seconds in first mode bending. Figure 10 is a plot of the predicted viscous hydrodynamic damping for the first bending mode as a function of current and significant wave height. Current is a much stronger factor than wave height.

In the thesis by Ghosh (2), several other interesting results are also presented. These include the effect of the sway motion of the guyed tower on the damping of the bending modes. Also included are predictions of dynamic respone as a function of seastate, and current profile.

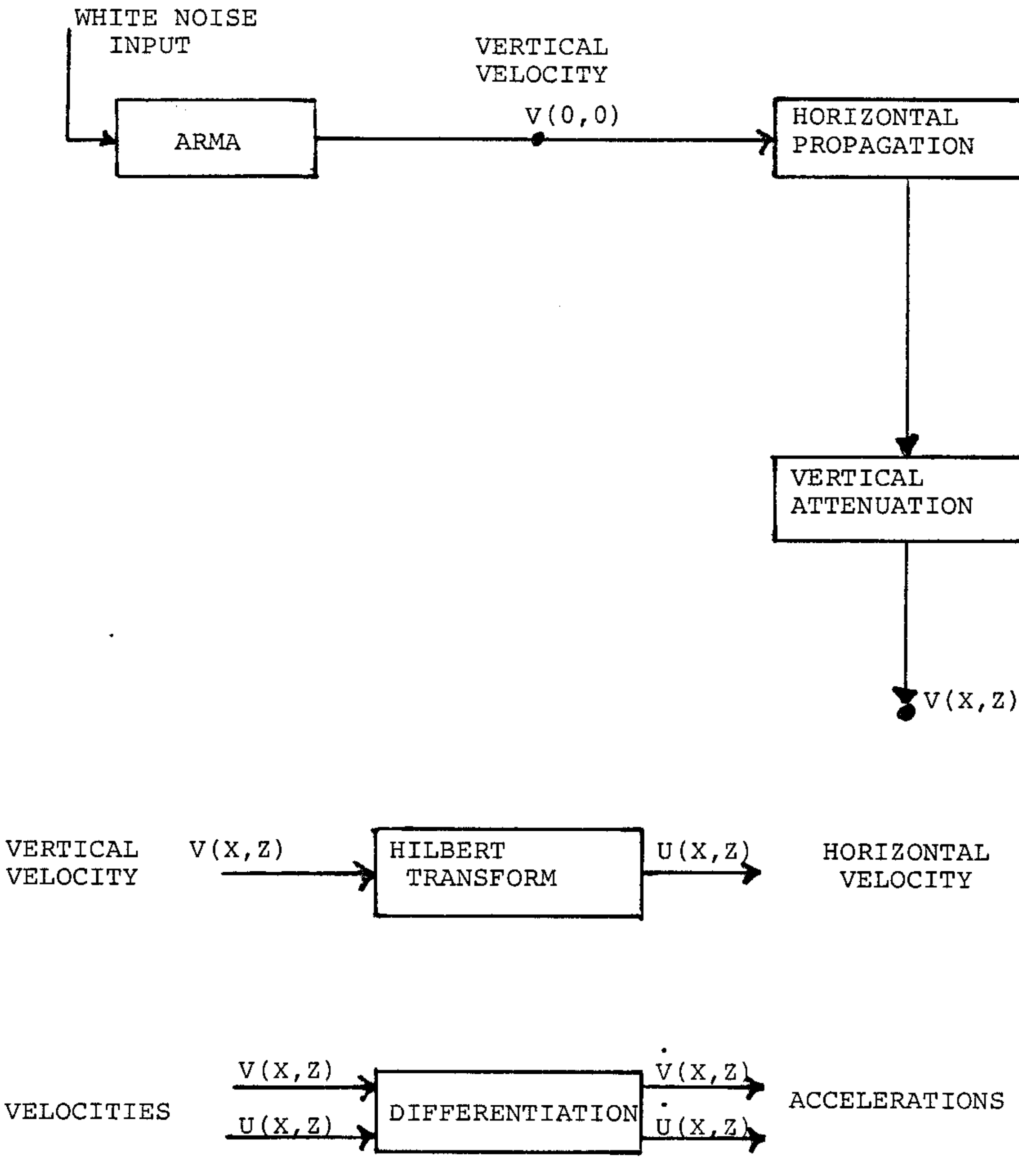



FIGURE 1  BLOCK DIAGRAM OF SIMULATION STEPS

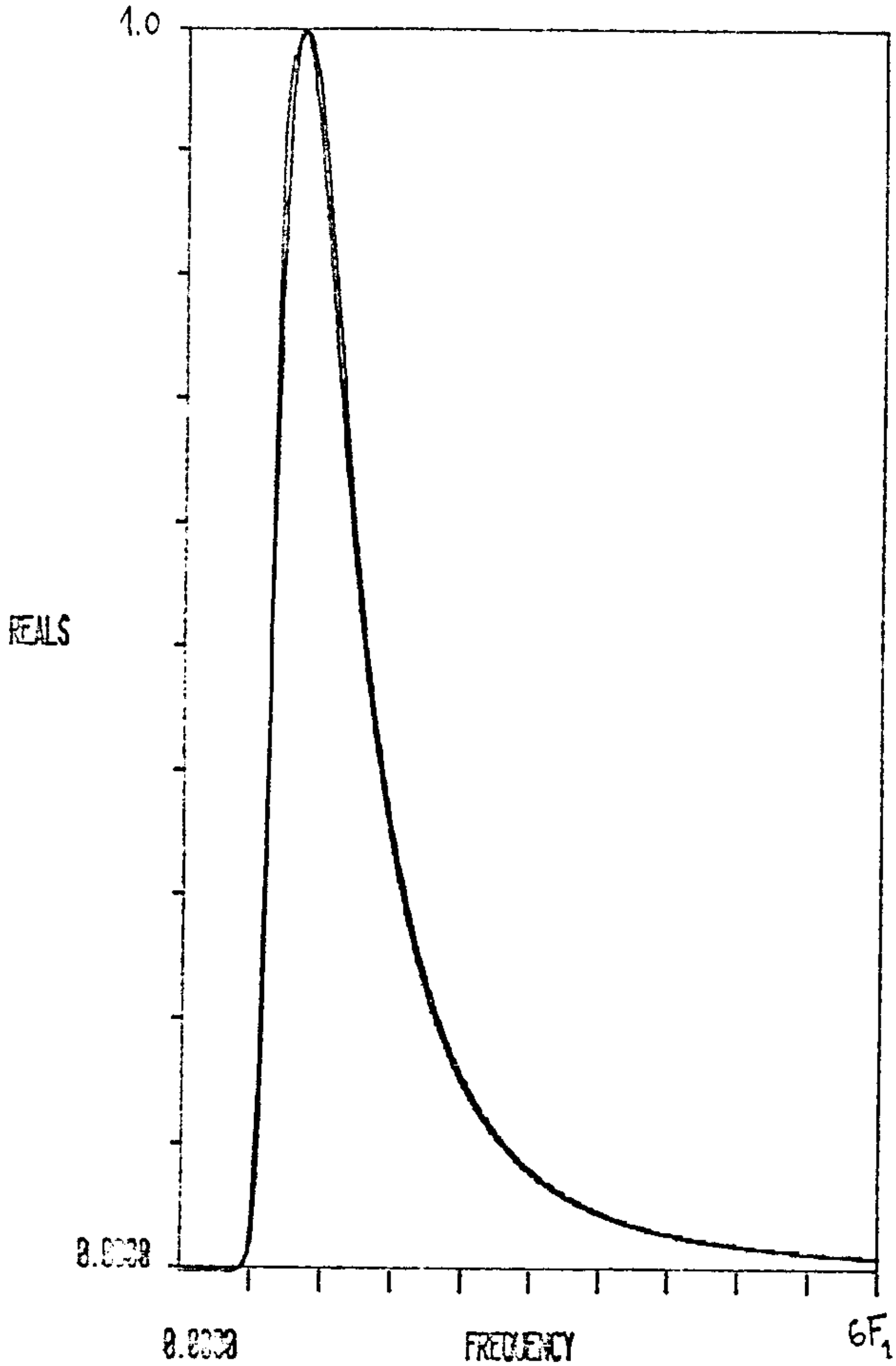



FIGURE 2

TARGET SPECTRUM AND ARMA FILTER TRANSFER FUNCTION

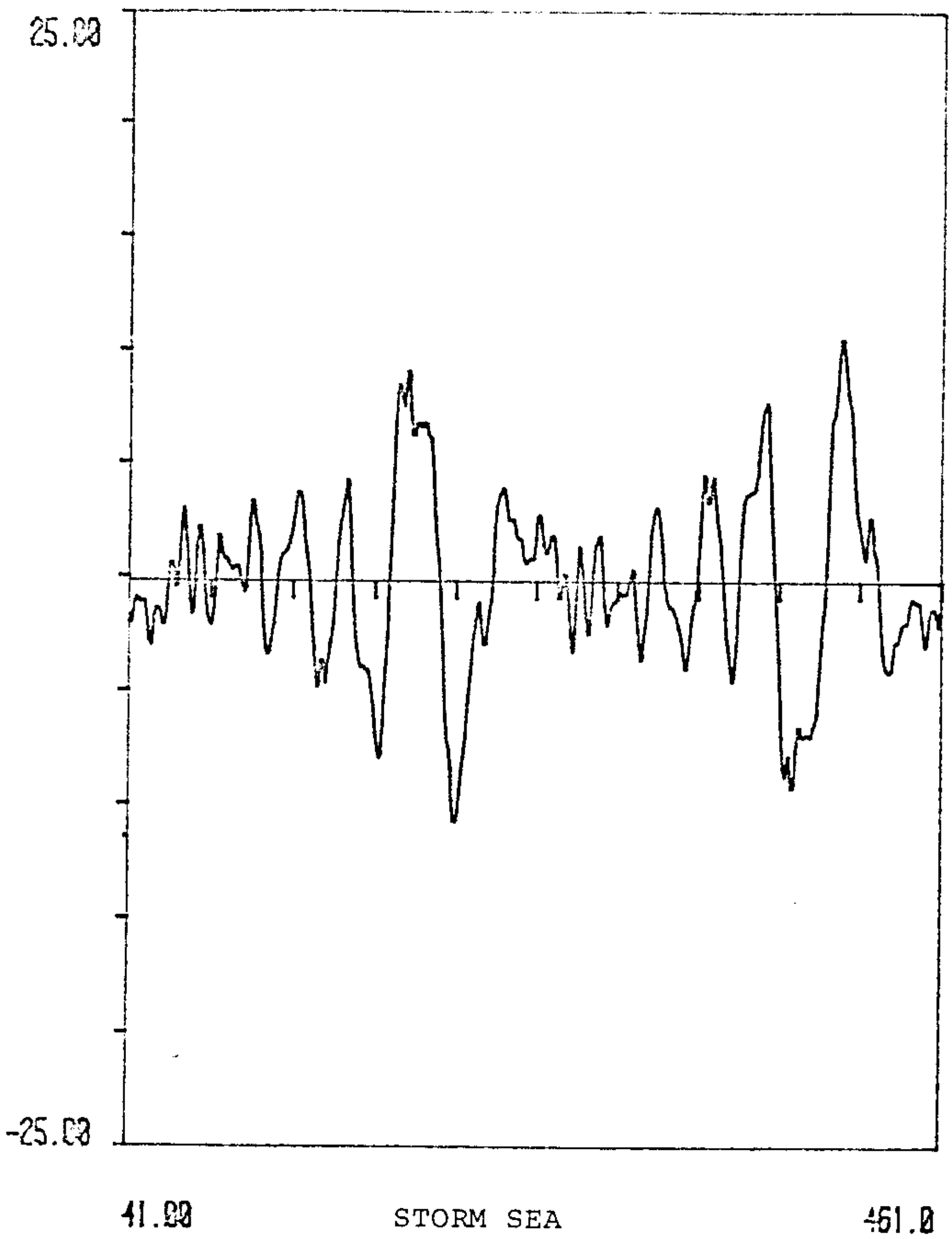



FIGURE 3

VERTICAL VELOCITY AT THE MEAN WATER LINE

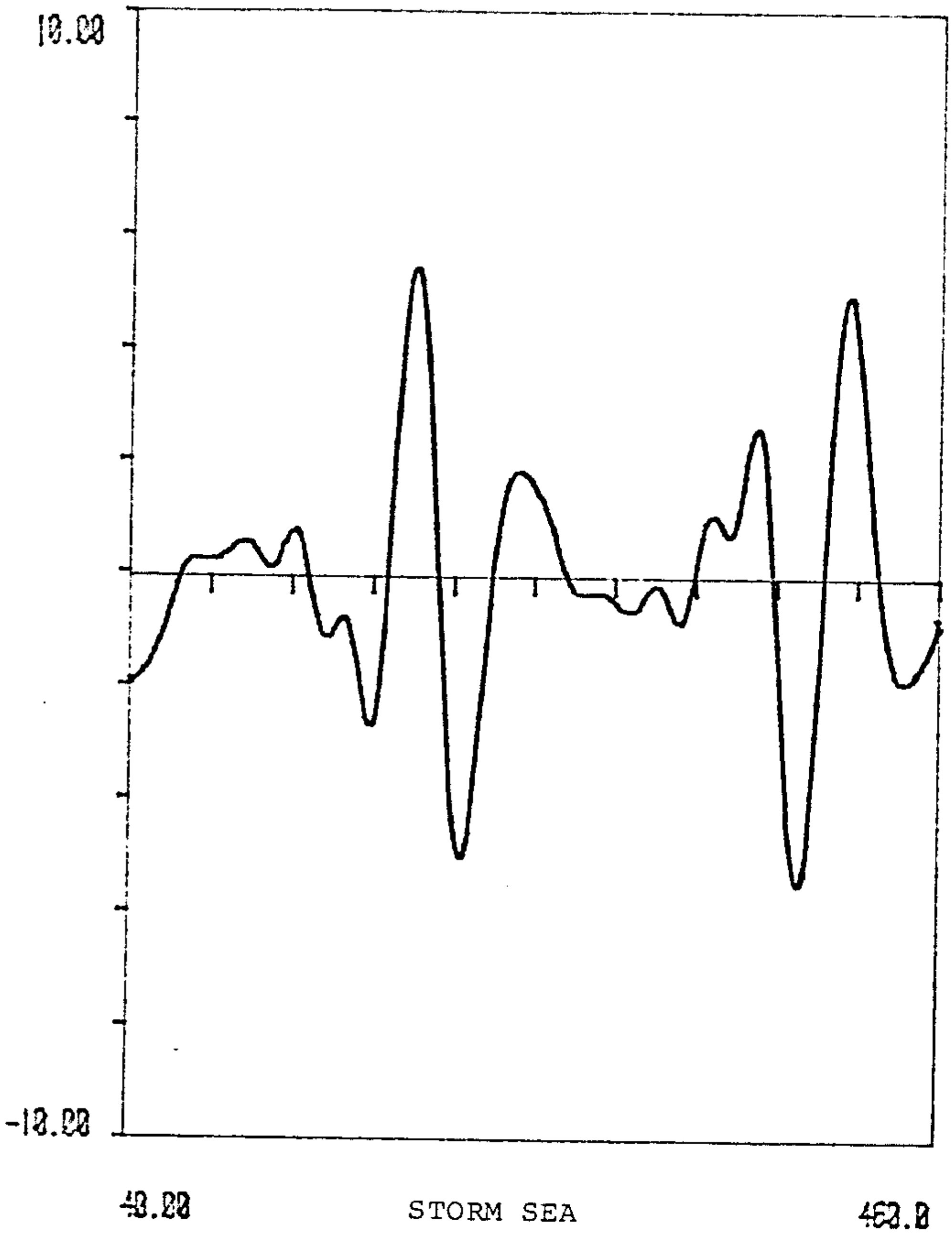



FIGURE 4

VERTICAL VELOCITY 20 METERS BELOW THE SURFACE -
EXACT AND SIMULATED

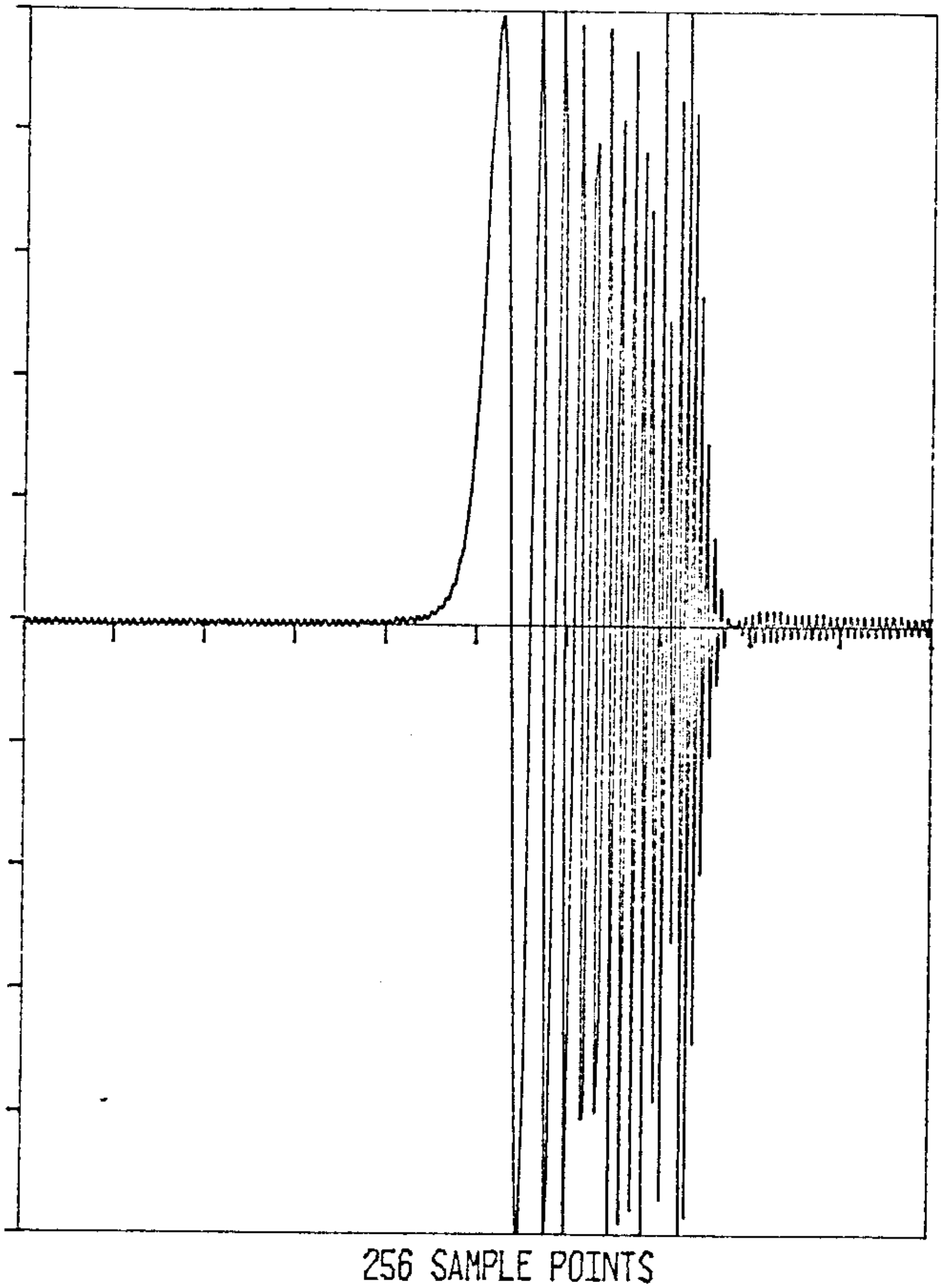
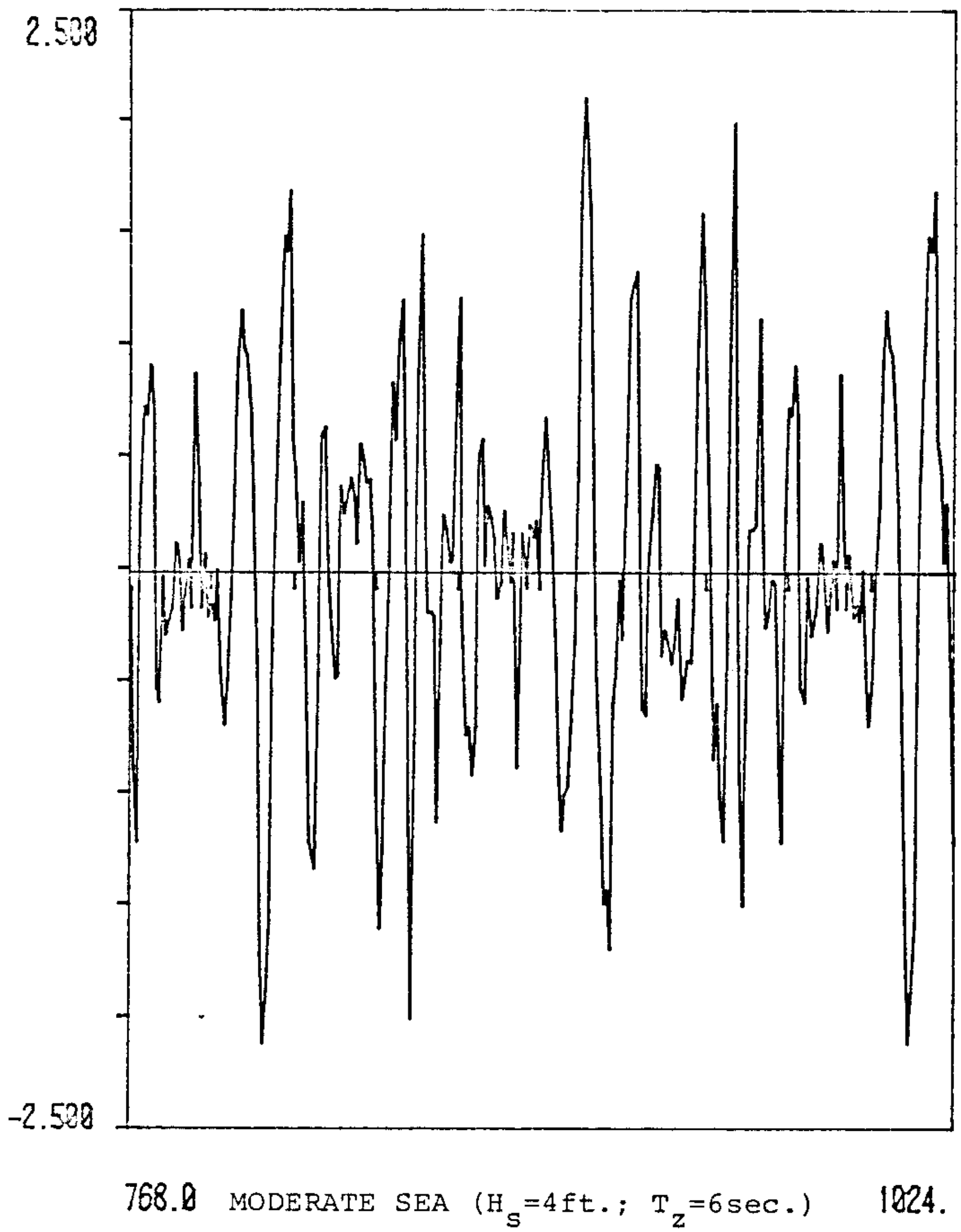



FIGURE 5

IMPULSE RESPONSE FOR HORIZONTAL PROPAGATION




MODERATE SEA ($H_s$=4ft.; $T_z$=6sec.)

FIGURE 6

EXACT AND SIMULATED TIME HISTORY
AFTER A 100M HORIZONTAL PROPAGATION

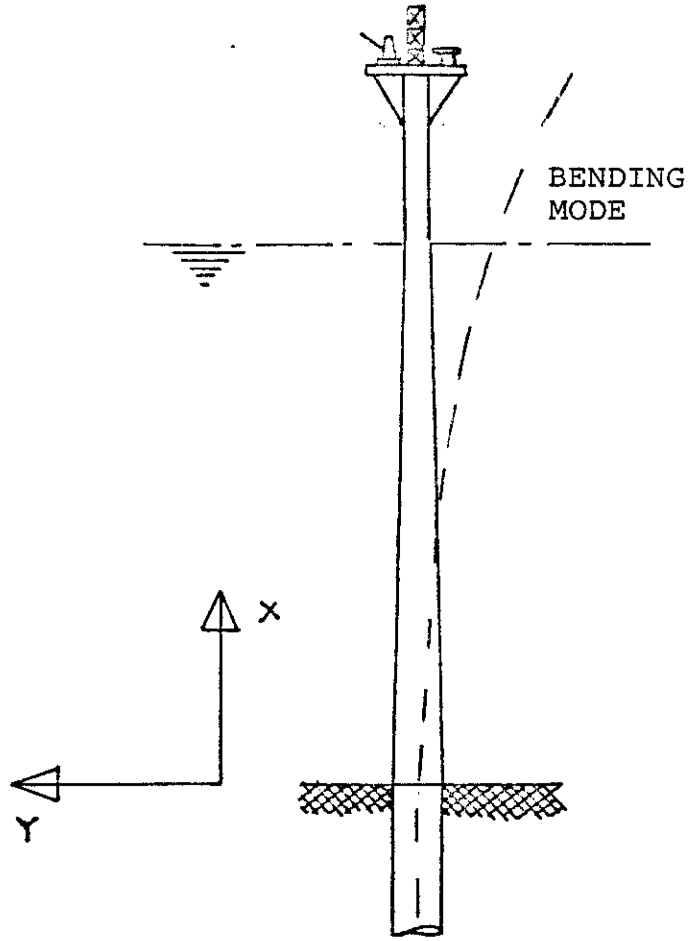



FIGURE 7 GULF OF MEXICO CAISSON

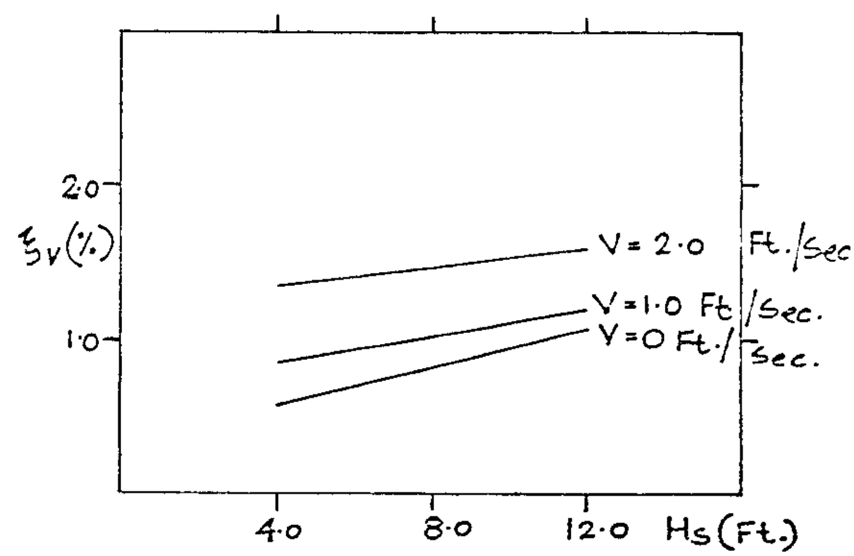



FIGURE 8 HYDRODYNAMIC VISCOUS DAMPING RATIO VERSUS SIGNIFICANT WAVE HEIGHT AND CURRENT

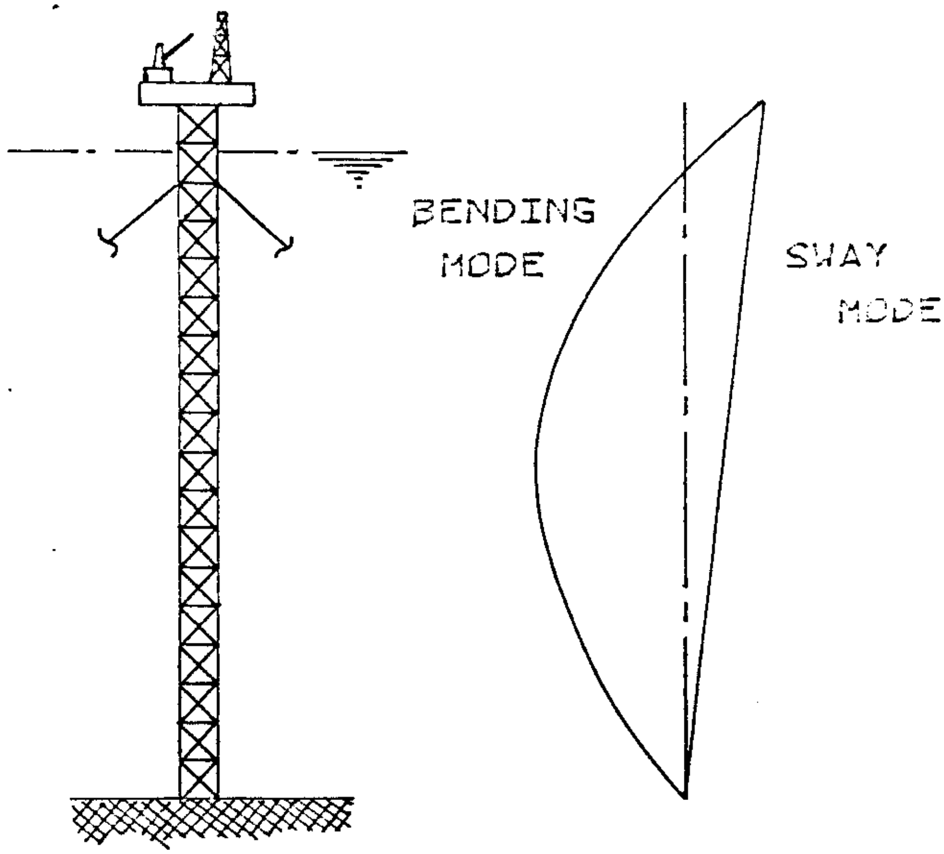



FIGURE 9 GUYED TOWER AND MODE SHAPES

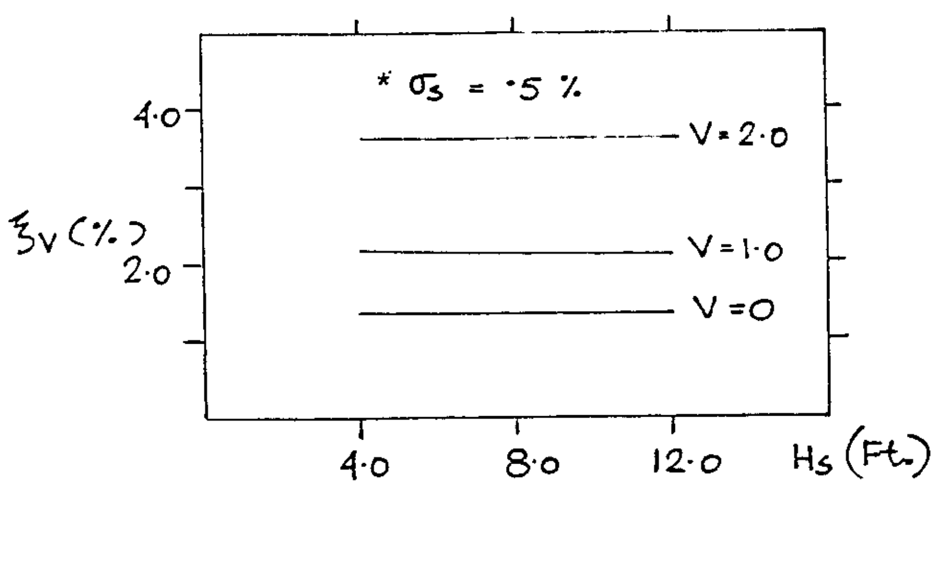



FIGURE 10 HYDRODYNAMIC VISCOUS DAMPING RATIO VERSUS SIGNIFICANT WAVE HEIGHT AND CURRENT

## REFERENCES


1. Samii, K., "Auto-Regressive Moving Average Simulations of Random Wave Forces on Offshore Structures," Ph.D. Dissertation, M.I.T., Department of Ocean Engineering, Cambridge, August 1983.

2. Ghosh, P.K., "The Prediction of Hydrodynamic Viscous Damping of Offshore Structures in Waves and Current by the Stochastic Linearization Method," S.M. Thesis, M.I.T., Department of Ocean Engineering, Cambridge, May 1983.