

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
DEPARTMENT OF OCEAN ENGINEERING

CAMBRIDGE, MASS. 02139

Room 5-222

January 5, 1978

Mr. John Gregory  
Research Program Manager  
Branch of Marine Oil and  
Gas Operations  
United States Geological Survey  
Department of the Interior  
620 National Center  
Reston, Virginia 22092

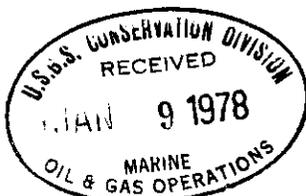
Dear John:

Two copies of my 11/21/77 report are enclosed. Please pardon the light copy. Also enclosed is a copy of my OTC paper. The part which describes how to evaluate the effect of liquid storage tanks on natural frequency is actually quite a recent breakthrough. Al Ruhl at Shell is already putting it to use.

To reiterate on our phone conversation, the grouted pile work is a masters thesis project by a Navy graduate student at no cost to USGS other than it diverts some of my time to advise him.

I have read the Aerospace report. They appear to be doing a credible and necessary portion of the work in evaluating the structural monitoring method. However, the project by Shell and 8 or 9 other companies to measure response on 3 large steel platforms is still very much alive, and I would hope that Aerospace doesn't duplicate that effort. Please let me know when their experimental results from South Pass 62C is available.

I am interested in receiving more information on their "truncated modes" method for finite element parametric damage studies, as it becomes available. Perhaps it is in the area of parametric studies of damage simulation that Aerospace can make an important contribution in the relatively near future. Two aspects of the problem mentioned in their report caught my attention. The first was improving computational efficiencies in the finite element program, and the second area is their computation of the ratio of flexural to torsional response as a function of damage.



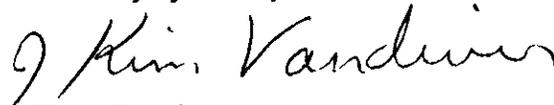
Mr. John Gregory  
January 5, 1978  
Page 2

To the extent they showed in their report, this seemed to be a promising indicator of damage in that it may be more sensitive than shifts in frequency to structural damage. Additional work in this area may bear fruit.

We have a good start in evaluating the effects of liquids and would hope that they would not duplicate our efforts there.

I will advise you on anything significant resulting from my trip to Houston.

Sincerely yours,

A handwritten signature in cursive script that reads "J. Kim Vandiver". The signature is written in dark ink and is positioned above the typed name.

J. Kim Vandiver  
Doherty Assistant Professor

JKV:clg

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
DEPARTMENT OF OCEAN ENGINEERING

CAMBRIDGE, MASS. 02139  
Room 5-222

November 21, 1977

Mr. John Gregory  
Research Program Manager  
Branch of Marine Oil and  
Gas Operations  
United States Geological Survey  
Department of the Interior  
620 National Center  
Reston, Virginia 22092

Dear Mr. Gregory:

The following is a letter report on our research for the 1st quarter as sponsored by USGS. Also provided is some detail on the direction our research will take during the remainder of the first year. Included is a description of work that is related to but not directly supported by the contract with USGS. This information is provided because our research in the second year under USGS sponsorship will bear upon all aspects of our current research.

Following this summary is a brief outline of the contents of my anticipated second year renewal proposal to be submitted to USGS during the spring of 1978.

1. First quarter accomplishments:

The accomplishments to date are mostly related to getting under way in an orderly fashion. The spectrum analyzer is expected to be delivered in mid January 1978. Because it plays a key role in much of the research the bulk of our effort has been timed to begin in January. Brad Campbell, a Ph.D. candidate will come on board as a full time research assistant at that time. His part time efforts this fall are preparatory in nature. By January he should have in mind the general thrusts of his thesis research. It will pertain to the analysis and prediction of dynamic response parameters of offshore structures, such as natural frequencies, mode shapes and damping. It will also include research on spectral analysis techniques, that are best suited to investigation of offshore structures. This fall he participated in dynamic response measurements on board the NAVFAC platforms near Cape Hatteras.

2. Anticipated accomplishments by 1 September 1978:

In the original proposal to USGS four topics were identified. They were:

- a. Identification and analysis of dynamic response properties.
- b. Artificial damping.
- c. Analysis of data from full scale structures.
- d. Prediction of dynamic response.

These topics are all related, and are areas in which we have experience. Some of our accomplishments in these areas this year will be due in part to support from sources other than USGS over the last four years. Obligations to all other sponsors will cease 30 June 1978.

- a. Identification and analysis of dynamic response properties.

As already mentioned, Brad Campbell, a Ph.D. student will concentrate in this area. His work which is as yet somewhat undefined, will begin in earnest in January 1978 and will extend over a period of at least two years.

In addition, an engineer from Shell recently posed the following problem. Some companies do not grout main leg piles. This introduces a non-linear stiffness that may affect the dynamic response. Does the presence of ungrouted piles decrease ones ability to obtain repeatable measures of natural frequency, thus decreasing the sensitivity of a structural monitoring program? During the spring of 1978 a theoretical study of this problem will be conducted as a masters thesis topic, by a naval officer graduate student in the department.

- b. Artificial damping.

The concept of using liquid storage tanks as dynamic dampers was introduced at the 6 October 1977 seminar at M.I.T. This work is continuing and will be presented at the May 1978 OTC.

As part of the work on the effects of grouting piles, we intend to investigate the possibility of artificially increasing damping by using grouting materials other than concrete. Furthermore, it is not clear if the practice of grouting improves or worsens the overall dynamic response properties of offshore structures. We hope to provide the designer with some helpful insight to this problem as well.

c. Full scale studies.

Through an agreement with Union Oil we will be provided with data on a platform before and after the piles were put in place. These data will be analyzed and the results integrated with our recent investigations on the effects of soil properties and scour on natural frequencies.

In late October 1977 we collected response data on 3 of the 4 ACMR platforms near Cape Hatteras. These platforms were erected during August 1977, are of similar construction and in similar sediments. They provide an unusual opportunity to observe early "settling in" effects of the soils on natural frequency and damping, and also provide an unusual statistical sample of 4 platforms of similar construction, in similar soils, and all of the same age. These structures are being instrumented by NAVFAC to telemeter structural response, wind, wave, and current data throughout the winter of 1978-1979. These data will be a considerable value to this research.

d. Prediction of dynamic response.

Since 1974, I have been developing a new method for the prediction of the dynamic response of ocean structures to random wave forces. This technique will be directly applicable to the problem of fatigue life prediction. A draft of a paper on the method has been recently completed and will be submitted for publication in the near future. In the coming year we hope to obtain preliminary verification of the theory by conducting model tests. We may also use the technique to make response predictions for a tension leg design currently under study by Shell.

3. Proposed research for the period 1 September 1978 through 30 August 1979:

The research will concentrate in two areas. The first is Brad Campbell's Ph.D. research, which as stated before will be on the topic of measurement and analysis of dynamic response parameters. One feature of this will be to examine the state of

the art methods for evaluating structural dynamic response, and then extend the state of the art as it applies to marine structures. A Ph.D. dissertation must advance the state of the art to be acceptable. The best available dynamic analysis software is a modal analysis program which sells for \$16,000 and is designed for use on our machine. The program, for example, will compute modal damping factors by three different techniques. It also computes natural frequencies and mode shapes, and animates the structural response on the CRT terminal. This graphics capability will be extremely useful in presenting in a visual way the results of our research. In the next few months Brad will complete a thesis prospectus that will be forwarded to you.

The second area of research will be the implementation and verification of our dynamic response prediction technique. The method will be applied to a variety of structures, including jacket type and tension leg platforms. Through the use of model tests and the evaluation of full scale data the method will be calibrated and improved. Telemetered wave height and structural response data from the NAVFAC platform will be extremely useful. This fall a new student has begun working for me, who I believe is Ph.D. material. By next fall he may be ready to begin Ph.D. research in this field. He is already working on the preliminary model tests and seems very capable.

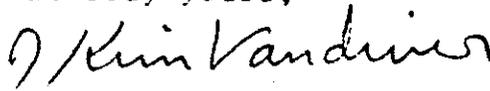
Eventually, this technique may prove to be an excellent early design stage tool, enabling the designer to simply and quickly identify the pertinent dynamic response features of his structure. The technique is applicable to a broad range of structures, both fixed and moored. In many cases it will replace the use of the Morrison wave force equation.

A tentative budget for the second year is outlined below.

Tentative Budget for 1 September 1978 - 30 August 1979

Principal Investigator J. Kim Vandiver (5 mm)	\$ 9,722
Secretary (3 mm)	2,400
Employee Benefits	3,152
Research Assistants (2) (24 mm)	21,480
Indirect Costs	17,169
Computer Software	16,000
Materials and Services	1,500
Travel	1,200
TOTAL	<u>\$ 72,623</u>

Sincerely yours,



J. Kim Vandiver  
Doherty Assistant Professor

JKV:clg

THE EFFECT OF LIQUID STORAGE  
TANKS ON THE DYNAMIC RESPONSE  
OF OFFSHORE PLATFORMS

by

J. Kim Vandiver  
and  
Shuhei Mitone

ABSTRACT

The sloshing of liquids in storage tanks on fixed offshore structures affects both the natural frequencies and damping. Analytic procedures by which one may account for these effects are presented. Also shown is a method for design of tankage that will result in suppression of dynamic response at the fundamental flexural natural frequencies of the structure. An important aspect is that no new equipment is required but only optimum configuration of tankage that is already required for storage of water, fuel, mud or crude oil.

## I. INTRODUCTION

The ability to accurately measure and predict the dynamic response properties of fixed offshore platforms is an important concern to the industry. In particular it is important to be able to measure and predict changes in the natural frequencies of the structure, and to measure and predict the damping of the structure at its natural frequencies.

It is known that structural damage will cause small shifts in the natural frequencies of the structure in flexure and torsion. However, detection of the damage by monitoring the changes in frequency will only be possible if all other sources of change are understood, predicted and accounted for. One important source of change in natural frequency that is not well understood is the dynamic response (sloshing) of liquids in storage tanks [1,2]. It has been observed that small changes in tank depth can result in substantial changes in response behavior. This paper provides a method by which the effects of liquid storage tanks on the flexural response of a structure may be predicted and accounted for.

The modal damping associated with each natural frequency is also important, because it controls the dynamic amplification at the flexural and torsional natural frequencies. This amplification must be considered when making fatigue life estimates for the structure.

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The damping on jacket structures is very low, typically 1 to 3% [1,3] and comes from a variety of sources: structural, viscous hydrodynamic, wave radiation, and soils. An additional and not insignificant source, which is generally overlooked, is the damping contributed by the sloshing of liquids stored in on board tanks.

This paper quantitatively assesses the damping contributed by the motions of stored liquids and furthermore provides the methods by which tankage design may be optimized to eliminate adverse dynamic properties and maximize the damping available to the structure.

The analysis in this paper has been restricted to the two orthogonal fundamental flexural modes of a structure with a rectangular planform. It is therefore applicable to most steel structures in the Gulf of Mexico and the North Sea. The fundamental flexural mode was chosen for analysis, because it is the most important contributor to dynamic response, and because it is most effected by deck level storage tanks. It is believed that the analysis may be extended to torsional modes, and higher order flexural modes as well.

## II. THE DYNAMIC RESPONSE MODEL OF A FUNDAMENTAL FLEXURAL MODE

By the method of modal analysis [4] the displacement response of the deck resulting from flexure of the platform in the fundamental mode, may be represented by an equivalent single degree of freedom oscillator, Figure 1. The required natural frequency, mode shape, and modal mass  $M$ , may be obtained from a finite element dynamics model of the structure. From these and a prescribed wave force spectrum a modal wave force spectrum may be derived. The response of this equivalent oscillator is such that it will have the same response energy as the actual flexural mode.

Furthermore, the quasi-static response of the deck displacement to large low frequency waves which correspond to the peak in the wave force spectrum will be reasonably well predicted by the motion of this equivalent oscillator, because the higher order modes respond very little to very low frequency wave forces.

The natural frequency and damping ratio of this oscillator are given respectively by,

$$\omega_o = \sqrt{\frac{K}{M}} \quad (1)$$

$$\zeta_o = \frac{C_o}{2\omega_o M} \quad (2)$$

where the dashpot constant  $C_o$  is chosen to yield the same damping ratio as the real structure. If  $X(t)$  and  $f(t)$  are the deck

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displacement and modal force time histories, with Fourier transforms  $X(\omega)$  and  $F(\omega)$ , then the complex frequency response of the oscillator is simply

$$\frac{X(\omega)}{F(\omega)} = H_X(\omega) = \frac{1/K}{\left[ \left(1 - \frac{\omega^2}{\omega_0^2}\right) + 2i\zeta_0 \frac{\omega}{\omega_0} \right]} \quad (3)$$

The response spectrum to a modal wave force spectrum is given by the well known relation:

$$S_X(\omega) = S_F(\omega) |H_X(\omega)|^2 \quad (4)$$

The mean square deck response may also be calculated from

$$E[X(t)^2] = \int_0^{\infty} S_X(\omega) d\omega \quad (5)$$

where for physically realizable systems we have allowed positive frequencies only.

This simple model does not include the effects of stored liquids. In the next section a model which is mechanically equivalent to the liquid sloshing motions is derived, and then joined to this system.

### III. THE SLOSHING MOTIONS OF A LIQUID IN AN OSCILLATING TANK

Figure 2 shows a rectangular tank, containing a liquid with a free surface. The tank is oscillated horizontally, by a displacement  $X(t) = A e^{i\omega t}$ . From potential flow theory, the horizontal forces exerted on the tank walls for the case of zero damping is given by:

$$F(t) = A\omega^2 e^{i\omega t} \left[ M_\ell + \sum_{n=1}^{\infty} \frac{8M_\ell \tanh \left[ (2n-1) \frac{\pi h}{a} \right]}{\frac{h}{a} (2n-1)^3 \pi^3 \left[ \frac{\omega_n^2}{\omega^2} - 1 \right]} \right] \quad (6)$$

where

$$\omega_n^2 = g(2n-1) \frac{\pi}{a} \tanh \left( \frac{2n-1}{a} \pi h \right) \quad (7)$$

and

$$n = 1, 2, 3, \dots$$

where

$\omega_n$  = nth natural sloshing frequency (rad/s)

$g$  = acceleration of gravity ( $m/s^2$ )

$a$  = tank length in direction of vibration (m)

$h$  = mean depth of liquid (m)

$M_\ell$  = total liquid mass (kg)

$r$  = aspect ratio =  $h/a$

the following has been assumed:

a. rigid rectangular tank

- b. homogeneous fluid
- c. incompressible fluid
- d. no sinks or sources
- e. small wave slopes
- f. zero damping

It is difficult to incorporate viscous effects in the potential flow solution. Even if one were able to do so, the result is difficult to combine with the single d.o.f. mechanical model of the flexural oscillations. A more useful formulation which may include viscous damping of the liquid, is an equivalent mechanical model of the sloshing. Figure 3 is a schematic of the mechanical equivalent to standing waves in an oscillating tank. The size of the equivalent masses may be determined by requiring that the force exerted on the tank wall by the equivalent mass-spring systems for the case of zero damping be equal to the forces calculated from potential flow theory.

The equation of motion for the nth equivalent mass is

$$m_n \ddot{x}_n + 2\zeta_n m_n \omega_n (\dot{x}_n - \dot{X}) + k_n (x_n - X) = 0 \text{ and} \quad (8)$$

$$\zeta_n = \frac{c_n}{2m_n \omega_n} \quad (9)$$

The coordinates  $x_n$  are taken relative to the equilibrium position of the masses when the tank is held fixed. For the case

of zero damping,  $c_n = 0$ , the force exerted by the vibrating masses on the walls of the tank when  $X(t) = A e^{i\omega t}$  is given by

$$F(t) = \left[ \sum_{n=1}^{\infty} k_n x_n \right] - m_o \ddot{X} \quad (10)$$

which after some manipulation results in the following:

$$F(t) = A \omega^2 e^{i\omega t} \left\{ \sum_{n=0}^{\infty} m_n + \sum_{n=1}^{\infty} \frac{m_n}{\frac{\omega_n^2}{\omega^2} - 1} \right\} \quad (11)$$

By comparing this to the forces in Equation 6, it is apparent that

$$\frac{m_n}{M_\ell} = \frac{8 \tanh\{(2n-1)\pi r\}}{\pi^3 r (2n-1)^3}, \quad n = 1, \infty \quad (12)$$

$$\sum_{n=0}^{\infty} \frac{m_n}{M_\ell} = 1 \quad (13)$$

$$m_o = M_\ell - \sum_{n=1}^{\infty} m_n \quad (14)$$

$m_o$  is the portion of the liquid which acts as a rigid body attached to the tank. It is not necessary to explicitly evaluate  $k_n$  because we require that

$$\omega_n^2 = \frac{k_n}{m_n} = \frac{g(2n-1)\pi}{a} \tanh \left( \frac{2n-1}{a} \pi h \right) \quad (1)$$

These derivations are presented in detail in references 5 and 6,

The response of the waves in the nth sloshing mode to either sinusoidal or random motions of the deck will be significant only in the frequency band near  $\omega_n$ , the natural frequency of that mode. Under these conditions it will be of interest to be able to calculate the wave amplitudes in the tank  $\alpha_n$  in terms of the deflections  $x_n$  of the equivalent mass. This may be done by requiring that the power dissipated by the damping be the same in each case. If the damping ratio  $\zeta_n$  is required to be the same for the liquid as the equivalent system, then  $\pi_n$ , the average power dissipated, is given by

$$\pi_n = 2\zeta_n \omega_n \langle E_n \rangle_{\text{liquid}} = 2\zeta_n \omega_n \langle E_n \rangle_{\text{mech}} \quad (15)$$

$$\therefore \langle E_n \rangle_{\text{liq.}} = \langle E_n \rangle_{\text{mech.}} \quad (16)$$

$\langle E_n \rangle$  denotes the average total energy of oscillation.

If we assume that due to dynamic amplification

$|x_n(t)| \gg |x(t)|$  and  $|\alpha_n(t)| \gg |x(t)|$ , then for the sloshing liquid with standing waves,

$$\langle E_n \rangle_{\text{liq.}} = \frac{\rho g}{2} E[\alpha_n^2(t)] \quad (17)$$

and for the mechanical equivalent

$$\langle E_n \rangle_{\text{mech.}} = K_n E[x_n(t)^2] = m_n \omega_n^2 E[x_n(t)^2] \quad (18)$$

where

$$E[\alpha_n(t)^2] \text{ and } E[x_n(t)^2]$$

are the mean square wave amplitude and mean square deflection, respectively. We have made use of the fact that for response dominated by resonance, the average kinetic energy equals the average potential energy, and therefore the average total energy is equal to twice the average potential energy. Equating the two results yields:

$$\frac{E[\alpha_n(t)^2]}{E[x_n(t)^2]} = \frac{2m_n \omega_n^2}{\rho g} = \frac{16 \tanh^2 [(2n-1)\pi r]}{(2n-1)^2 \pi^2} \quad (19)$$

The ratio of the root mean squares of the wave amplitude to the mass deflection is.

$$\frac{\sigma_{\alpha_n}}{\sigma_{x_n}} = \frac{4}{(2n-1)\pi} \tanh [(2n-1)\pi r] \quad (20)$$

It is noted that both  $\alpha_n(t)$  and  $x_n(t)$  have zero mean. Of course, if the excitation is sinusoidal at  $\omega_n$ , then the ratio of the amplitudes  $|\alpha_n|/|x_n|$  is also given by Equation 20. The mechanical equivalent sloshing model may now be combined with a dynamic platform model.

#### IV. PLATFORM RESPONSE WITH LIQUID STORAGE TANKS

In Section II a fundamental flexural mode was modeled by an equivalent single degree of freedom oscillator. That model may be altered as shown in Figure 4 to account for the liquid motions. The mechanical equivalent masses, springs and dashpots are simply attached in parallel to the single d.o.f. model. The system is no longer a simple single degree of freedom one, but in fact has  $j+1$  degrees of freedom, where  $j$  is the number of tank modes included in the analysis. The equivalent masses  $m_n$ , grow smaller as  $1/(2n+1)^3$ , and therefore the series may be truncated after a very few terms. However, all terms must be included for which the natural frequencies of the tank  $\omega_n$  are less than or equal to the flexural natural frequency  $\omega_0$ . The first term for which  $\omega_n > \omega_0$  might also be included, but thereafter the series may be truncated. Under no circumstances should terms be kept whose frequencies  $\omega_n$  approach the 2nd flexural natural frequency, because the resulting predicted response will be totally erroneous at these frequencies. The total mass of the truncated oscillators should be added to the modal mass of the platform. This is reasonable, because the deck motion  $X(t)$  will be dominated by motion near and below  $\omega_0$ , the nominal flexural fundamental frequency. Hence, the masses with natural frequencies higher than this will move in phase with  $X(t)$  and with approximately equal amplitude. The equation of motion for each of the

equivalent liquid masses is given by Equation 8. The equation of motion for the deck motions in flexure for frequencies up to and including the fundamental flexural mode is given by

$$F(t) = M_T \ddot{X} + C \dot{X} + KX + \sum_{n=1}^j [k_n (X - x_n) + C_n (\dot{X} - \dot{x}_n)] \quad (21)$$

where

$$M_T = M + M_L - \sum_{n=1}^j m_n \quad (22)$$

Equations 8 and 21 form a set of  $j+1$  simultaneous equations. For the homogeneous case with zero damping these equations reduce to an eigenvalue problem of the following familiar form, which may be solved for the  $j+1$  natural frequencies of the system.

$$-\omega^2 [M] \begin{Bmatrix} X \\ x_1 \\ \vdots \\ x_j \end{Bmatrix} + [K] \begin{Bmatrix} X \\ x_1 \\ \vdots \\ x_j \end{Bmatrix} = 0 \quad (23)$$

This may be solved for the  $j+1$  natural frequencies and mode shapes of the system. In this way one may directly calculate the shift in natural frequency of the fundamental flexural mode, as a function of liquid on board. It must be noted that the lowest natural frequency of this system is no longer necessarily the fundamental flexural mode. In fact if  $\omega_1$  the first tank

mode is lower than the  $\omega_0$ , the flexural frequency with no liquids on board, then the lowest natural frequency of the system will more closely correspond to  $\omega_1$ . The measured displacement response spectrum for such a structure may have a peak at a frequency near  $\omega_1$ . This was in fact observed by A. Ruhl on the shell platform South Pass 62C. [3] Furthermore, due to the presence of the liquid the nominal flexural frequency  $\omega_0$  will now be shifted by an amount which can be determined from the eigenvalue solution.

The complex frequency response for this system may be derived from Equation 21 and is

$$\frac{X(\omega)}{F(\omega)} = H_x(\omega) = \frac{1}{(-\omega^2 M_T + i\omega C_0 + K) - \omega^2 \sum_{n=1}^j m_n \frac{i\omega c_n + k_n}{-\omega^2 m_n + i\omega c_n + k_n}} \quad (24)$$

If this expression is used in Equation 4 then one may solve for the deck displacement response spectrum directly. The additional spectral peaks, and the shift in the peak of the flexural fundamental will be explicitly predicted. If this expression is integrated as in Equation 5 then the effect of the tank on the total mean square response may be evaluated.

A weakness of this analysis is that it assumes that the single degree of freedom model of the fundamental flexural mode obtained by modal analysis of the structure without tanks remains a valid representation of the fundamental flexural mode after the tanks are added on. This is a reasonable assumption if the fundamental flexural natural frequency and mode shape are not altered greatly by the presence of the tanks. Even if they are the qualitative insight gained from the analysis will be correct, but accurate quantitative information may require a modal analysis which includes from the beginning a mechanical equivalent model of the tank. In any case this method is far superior to the approximate corrections previously suggested by the author [1,2].

The remainder of this paper addresses a special case in which the tank is designed to suppress the total mean square response of the fundamental flexural mode in an optimum way.

#### V. SUPPRESSION OF DYNAMIC RESPONSE

As we noted in the previous section, the deck displacement response spectrum may have a noticeable peak near  $\omega_1$ , the lowest sloshing frequency in the tank. This additional peak increases the total mean square response of the deck. However, if the tank is designed such that  $\omega_1 \approx \omega_0$  then the total mean square response may in fact be reduced. In the jargon of the mechanical vibration literature, the tank behaves as a dynamic absorber [7].

To simplify the analysis we shall assume that  $\omega_1 \approx \omega_0$  and that the effect of the higher tank modes on the deck response near and below  $\omega_0$  may be ignored. As was described earlier the mass of the higher modes,  $\sum_{n=2}^{\infty} m_n$ , will be added to the modal mass of the platform. With this simplification the system reduces to that shown in Figure 5, a diagram of the equivalent two degree of freedom system.

The complex frequency response for  $X(t)$  the deck displacements for this system is given by

$$H_x(\omega) = \frac{-\omega^2 m_1 + i\omega c_1 + k_1}{(-\omega^2 M_T + i\omega C_0 + K)(-\omega^2 m_1 + i\omega c_1 + k_1) - \omega^2 (i\omega c_1 + k_1) m_1} \quad (25)$$

For the case that the first natural frequency of the tank  $\omega_1$  and the fundamental flexural frequency of the tower are approximately equal  $|H_x(\omega)|$  has two closely spaced peaks. It is noted that for this analysis the following definitions apply,

$$\omega_1 = \sqrt{\frac{k_1}{m_1}} \quad (26)$$

$$\omega_0 = \sqrt{\frac{K}{M + M_L - m_1}} = \sqrt{\frac{K}{M_T}} \quad (27)$$

The properties of the two peaks are dependent on  $\omega_1/\omega_0$ ,  $\zeta_0$ ,  $\zeta_1$ , and  $\mu$  the mass ratio  $m_1/M_T$ . The damping ratio  $\zeta_0$  is given by

$$\zeta_0 = \frac{c_0}{2\omega_0 M_T}, \text{ and} \quad (28)$$

is essentially equal to the damping ratio of the structure without liquids on board.

$$\zeta_1 = \frac{c_1}{2\omega_1 m_1} \quad (29)$$

$$\mu = \frac{m_1}{M_T} \quad (30)$$

The response spectrum from Equation 4 is

$$S_x(\omega) = S_F(\omega) |H_x(\omega)|^2$$

If for example we assume a Pierson-Moskowitz wave amplitude spectrum and calculate a wave force spectrum using Airy wave theory, then a typical response spectrum  $S_x(\omega)$  might be as shown in Figure 6. Two cases are shown. The case  $\mu = 0$  corresponds to no tank at all, and the second case is for  $\mu$  greater than zero but much less than one and  $\omega_1 \approx \omega_0$ . The important features are that the low frequency quasi-static response is not

affected by the presence of the tank, but that the tank causes the single resonant peak to split into two peaks of typically unequal height. For a given  $\mu$ , the relative height of the peaks will depend on the exact ratio of  $\omega_1/\omega_0$  and upon the damping  $\zeta_1$  and  $\zeta_0$ . The optimum response for a given  $\mu$  will be that which minimizes the area under the response spectrum in the dynamic region. This is best illustrated by example.

#### VI. EXAMPLE OF A 300 METER PLATFORM

The fundamental flexural mode is assumed to be  $f_0 = \omega_0/2\pi = 0.2$  Hz (5 sec period), and the damping from all sources other than the tank is assumed to be  $\zeta_0 = .03$ . The relative height of the two resonant peaks may be adjusted by small adjustments to the ratio  $\omega_1/\omega_0$ . In practice this involves changing the depth to length ratio  $r = h/a$  for a tank of given total liquid mass  $M_L$ . The optimum choice is to make the two peaks of equal height. This is most easily done by a few iterations on the computer. An initial approximation to the optimum frequency ratio  $\omega_1/\omega_0$  is given by [7]

$$\frac{\omega_1}{\omega_0} \approx \frac{1}{1 + \mu} \tag{31}$$

Once peaks of equal height have been achieved through variation of  $\omega_1/\omega_0$ , then the total dynamic response may be minimized by varying the damping ratio  $\zeta_1$  of the liquid. It is assumed that  $\zeta_0$  is essentially fixed. Figures 7 and 8 are sample plots of the dynamic response region of  $S_x(\omega)$  for two different values of  $\mu$  and several values of  $\zeta_1$ , where  $\omega_1/\omega_0$  has been adjusted to achieve peaks of equal height.

Figure 9 presents the mean square response in the dynamic region as a function of  $\mu$  and  $\zeta_1$  for the case of equal height optimum tuning of the peaks. The mean square dynamic response is normalized by dividing by the mean square dynamic response with no tank (i.e.  $\mu = 0$ ). For this example the dynamic response region was defined for  $f \geq 0.12$  Hz. Therefore

$$E[x(t)^2]_{\text{dyn.}} = \int_{(2\pi)(0.12 \text{ Hz})}^{\infty} S_x(\omega) d\omega \quad (32)$$

From Figure 9 it is clear that as the mass ratio  $\mu$  increases the total mean square response decreases. Hence large tanks provide better suppression of motion. Furthermore, for any particular mass ratio there exists an optimum value of liquid damping. The liquid damping may be adjusted by varying the viscosity of the liquid, or by adding screens and baffles to

the tank. Figure 9 illustrates that substantial reductions in mean square dynamic response may be achieved by careful selection of tank dimensions and damping.

## VII. CONCLUSIONS

The sloshing motions of liquids in tanks may alter both the measured natural frequencies of a platform as well as the damping. In the particular case that the lowest mode of the tank is very near the flexural fundamental a splitting of the response peak will occur and the nature of the split peak will vary with small changes in the depth of the tank. To a lesser extent similar phenomena may occur if higher tank modes are tuned to structural natural frequencies. The analysis was shown for the fundamental flexural mode but similar behavior is expected for torsional and higher order flexural motions as well.

Even though the industry is capable of designing safe structures without consideration of the effects of the motions of liquids, the analysis shows that dynamic response may be increased by the presence of very low frequency tank modes. Furthermore, by prudent selection of tank geometry, platform response may be reduced by using the tank as a dynamic absorber. Since on board tanks are a necessity they may as well be designed in such a way that their properties contribute favorably to the overall dynamic response performance.

### VIII. NOMENCLATURE

$\omega$	circular frequency, radians/s
$\omega_0$	natural frequency of single degree of freedom oscillator
$K$	spring constant of single d.o.f. system
$M$	mass of single d.o.f. system
$C_0$	dashpot constant of single d.o.f. system
$\zeta_0$	damping ratio for single d.o.f. system
$\omega_n$	nth sloshing natural frequency
$k_n$	nth equivalent spring constant
$m_n$	nth equivalent mass
$c_n$	nth equivalent dashpot constant
$\zeta_n$	nth damping ratio
$X(t)$	displacement of deck and single d.o.f. oscillator
$f(t)$	modal exciting force
$X(\omega), F(\omega)$	Fourier transforms of $X(t)$ and $f(t)$
$S_x(\omega),$ $S_f(\omega)$	power spectra of $X(t)$ and $f(t)$
$H_x(\omega)$	complex frequency response
$F(t)$	fluid force on tank walls
$M_l$	total liquid mass
$h, a$	depth and length of tank
$\langle E_n \rangle$	average energy of vibration of nth mode at resonance
$\pi_n$	power dissipated by nth sloshing mode at resonance
$\alpha_n(t)$	wave amplitude

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$\rho$  density of liquid in tank  
 $\mu$  mass ratio of dynamic absorber to platform modal mass  
 $M_T$  total modal mass of platform

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## IX. ACKNOWLEDGMENTS

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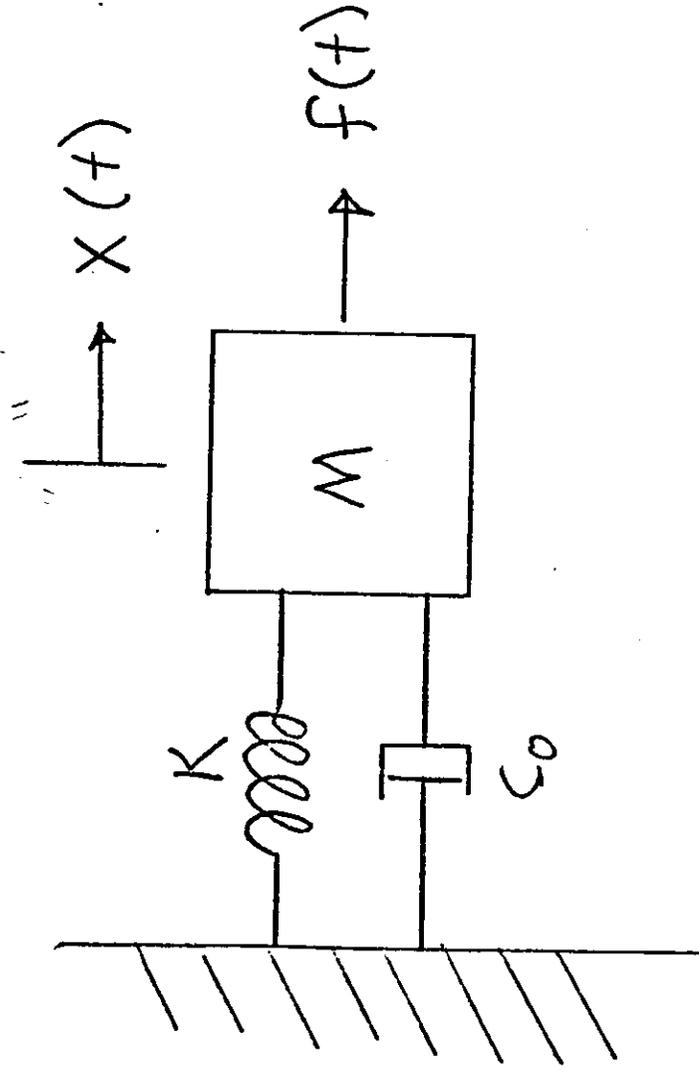


Figure 1. Single Degree of Freedom Equivalent to the Fundamental Flexural Mode.

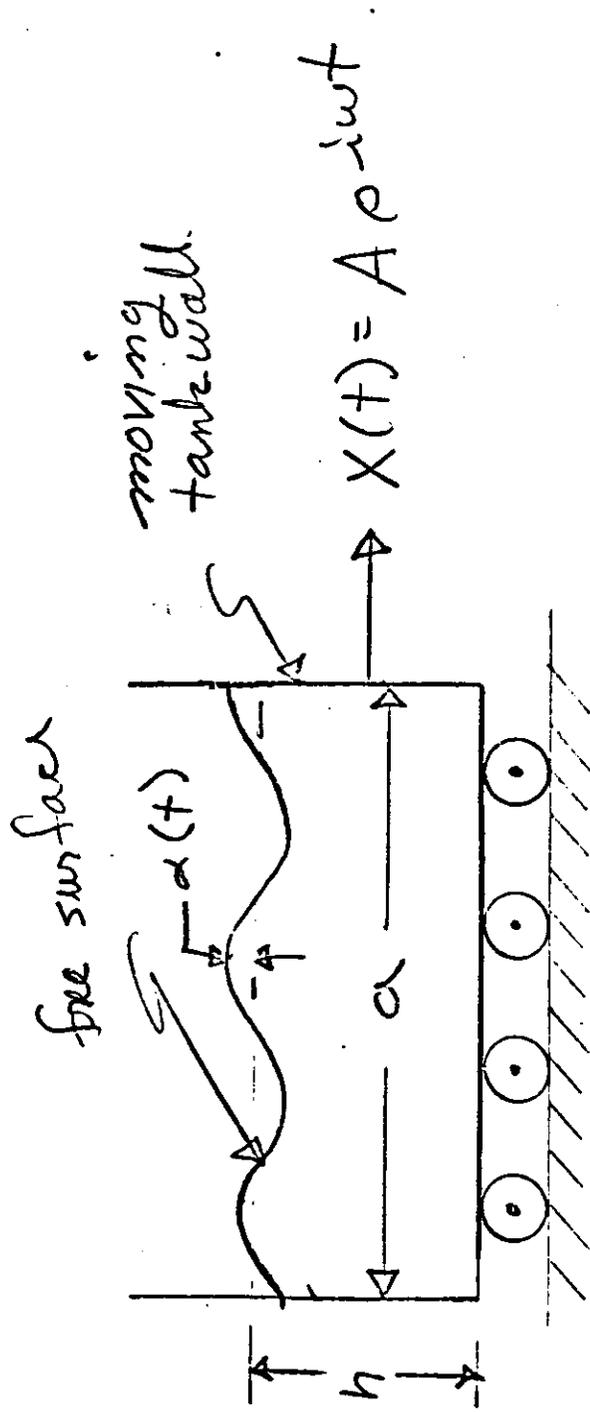


Figure 2, Oscillating Rectangular Tank with standing waves

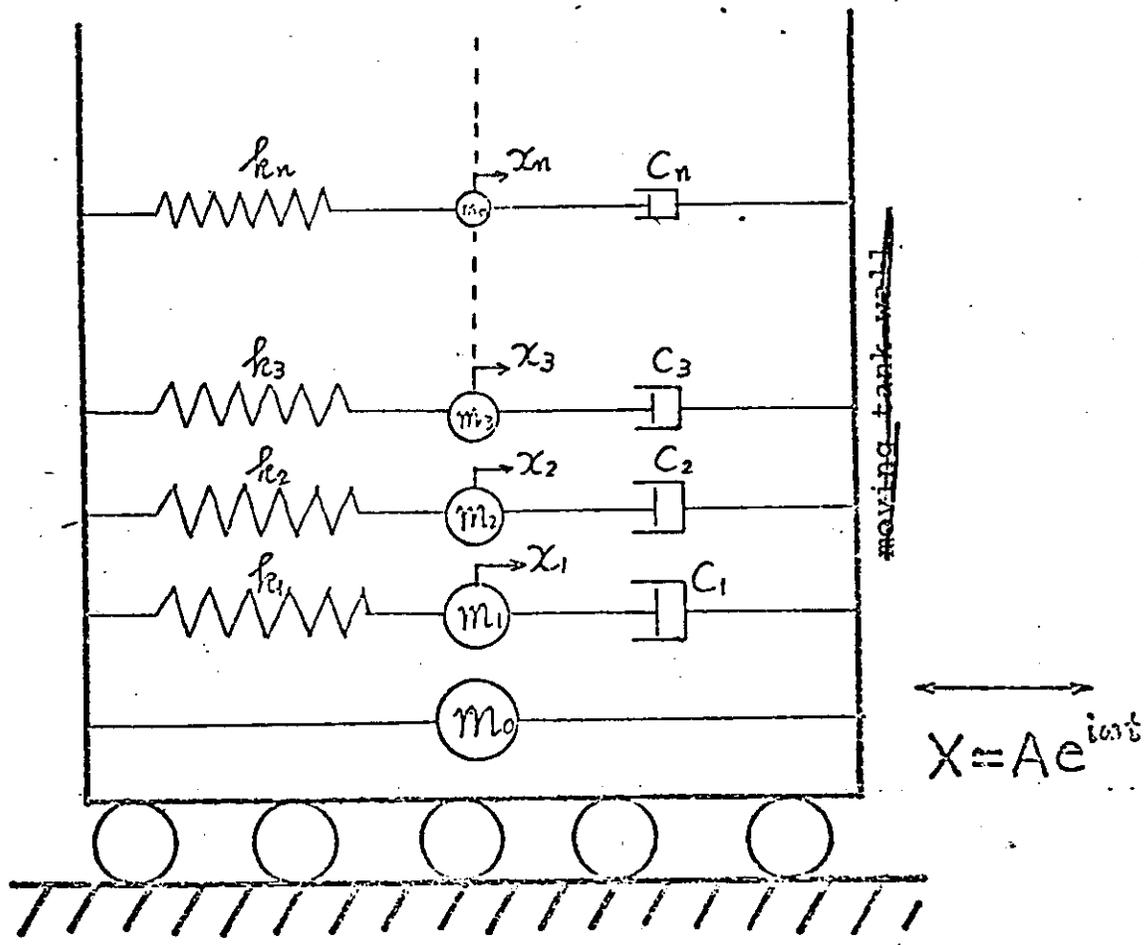
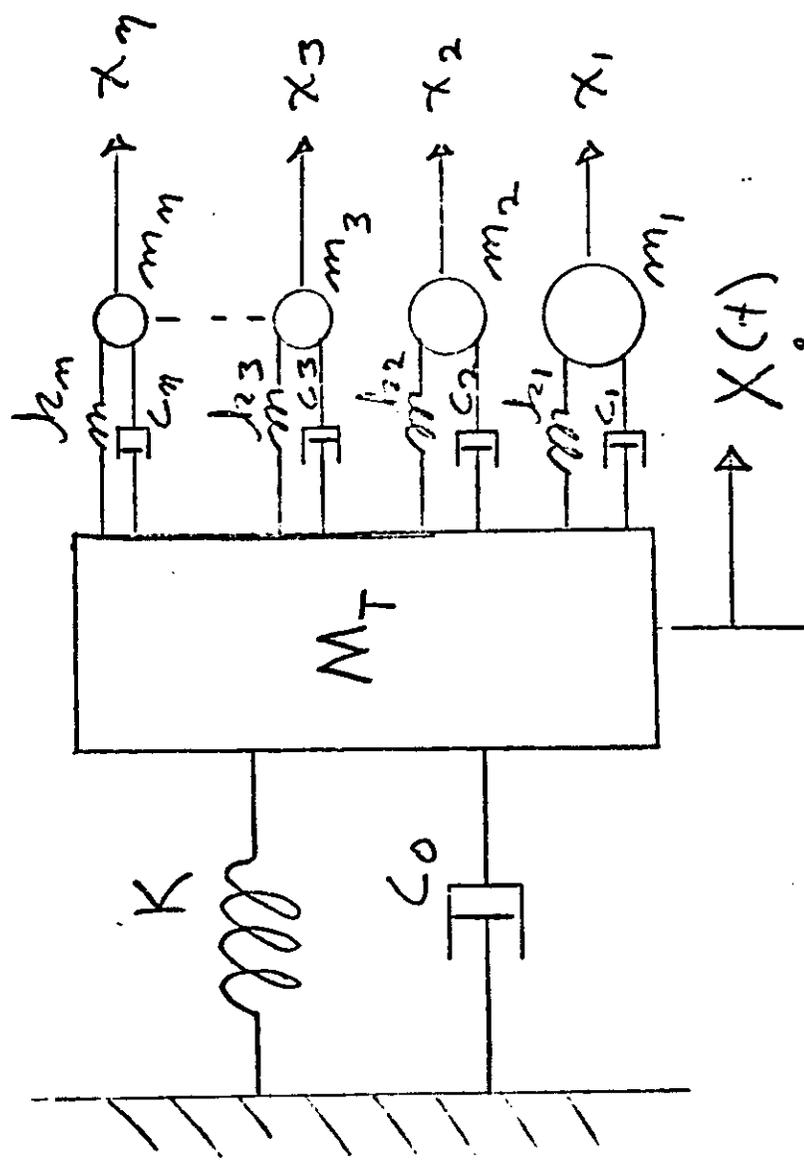


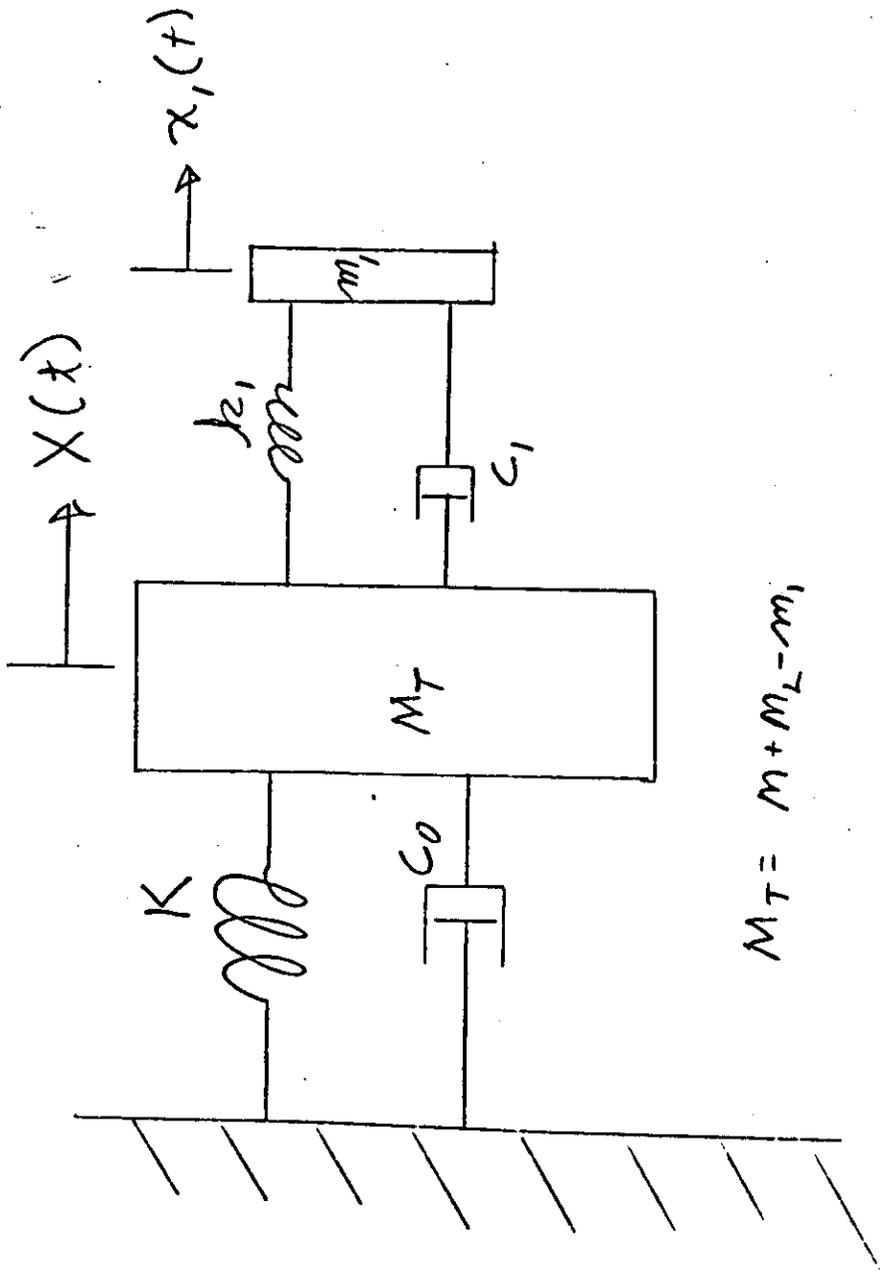
Fig. 3 Equivalent Mechanical Model of Sloshing

~~FIG. 3-2 MECHANICAL MODEL OF SLOSHING WITH VISCOUS DAMPER  
 (Coordinate of  $x_i$  measured at the equilibrium points of masses)~~



$$M_T = M + M_L - \sum_{n=1}^j m_n$$

Figure 4 Equivalent model of platform and Tank



$$M_T = m + m_L + m_1$$

Figure 5. Platform Model with a Dynamic Absorber

$$S_x(f) / S_x(f)_{\max} \equiv \overline{S_x(f)}$$

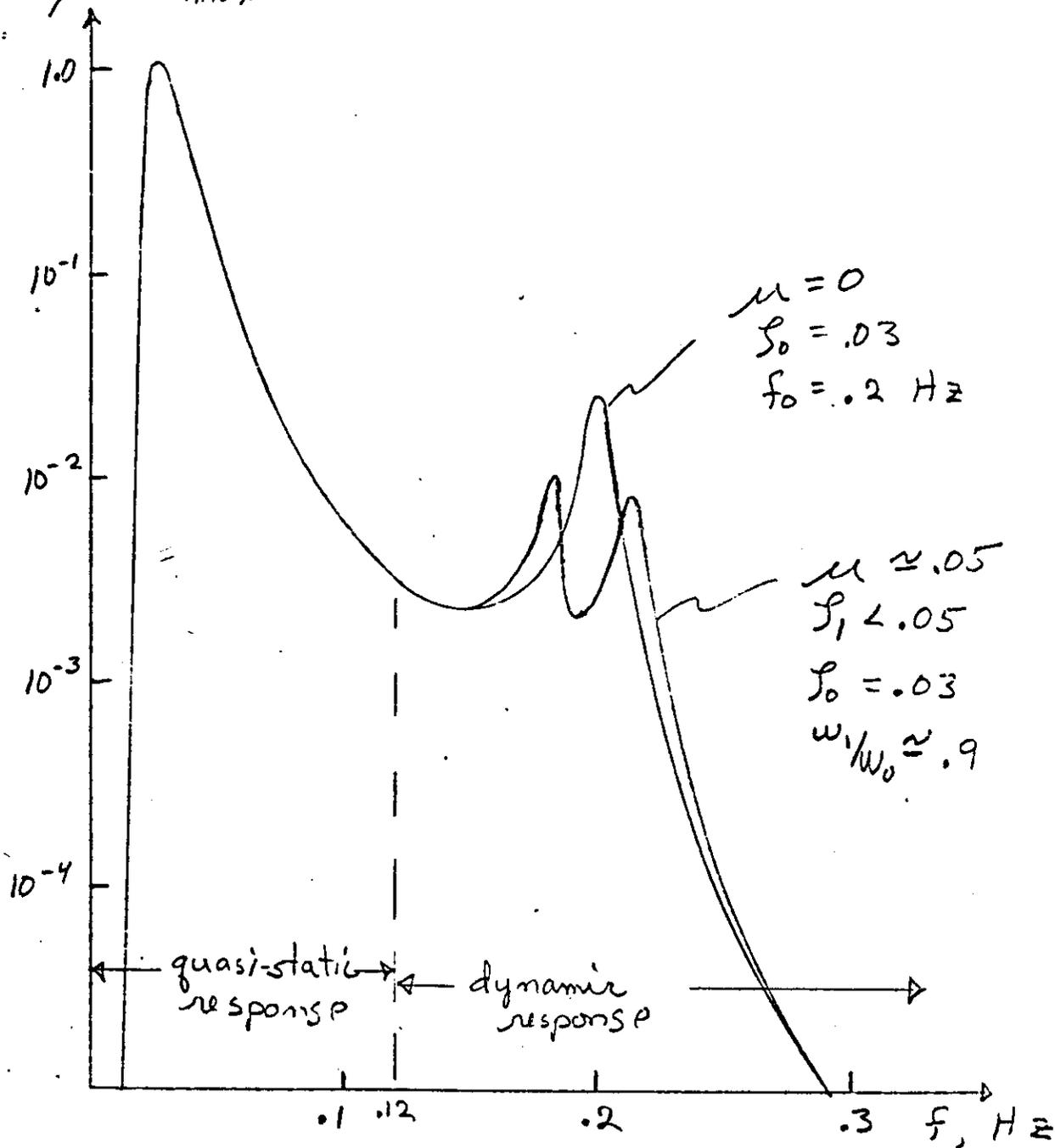


Figure 6. Typical Response Spectrum with a Tuned Tank

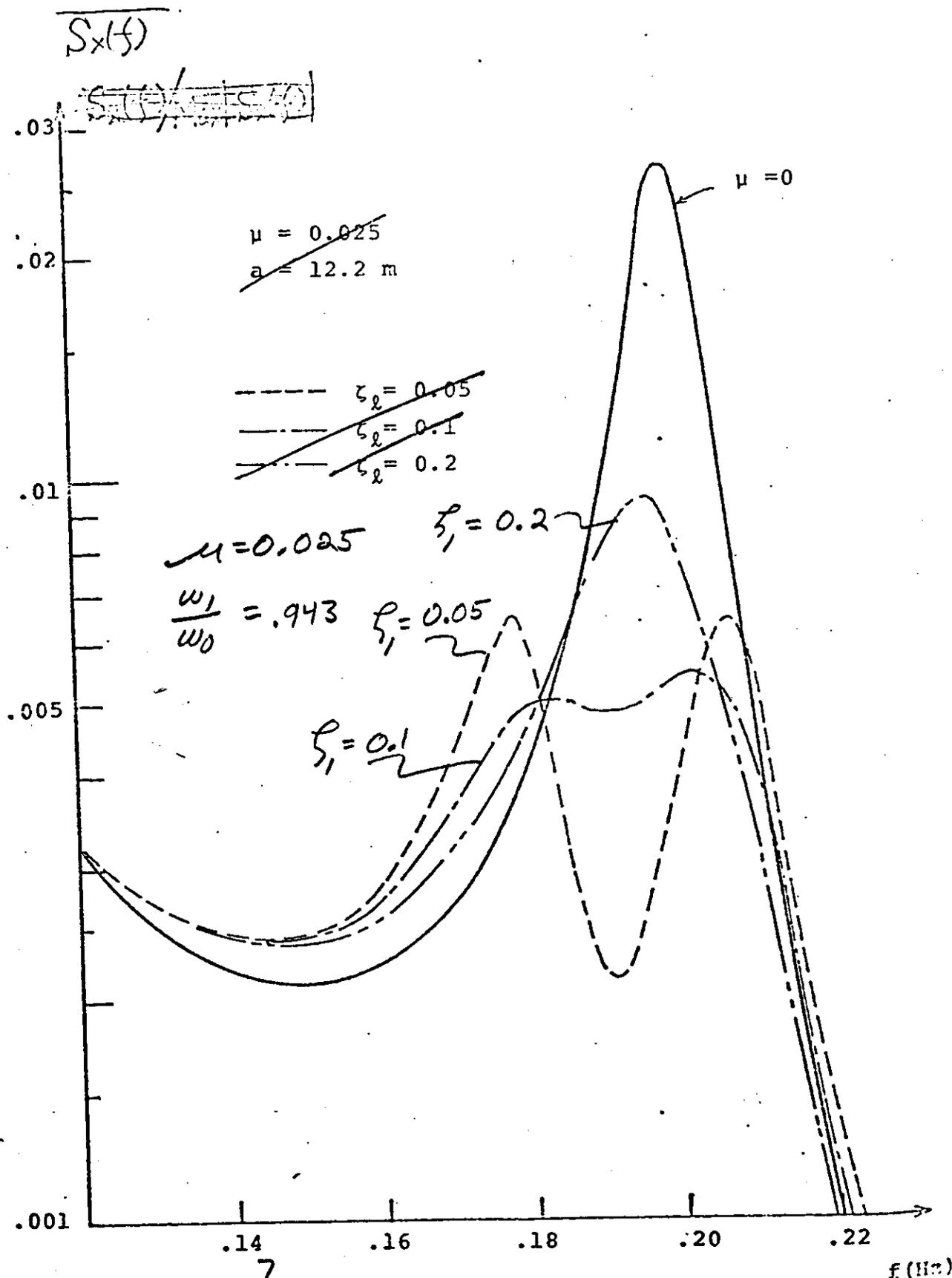


FIG. 7 RESPONSE DEFLECTION SPECTRUM OF 300 M PLATFORM ~~Example 10~~ Example 10

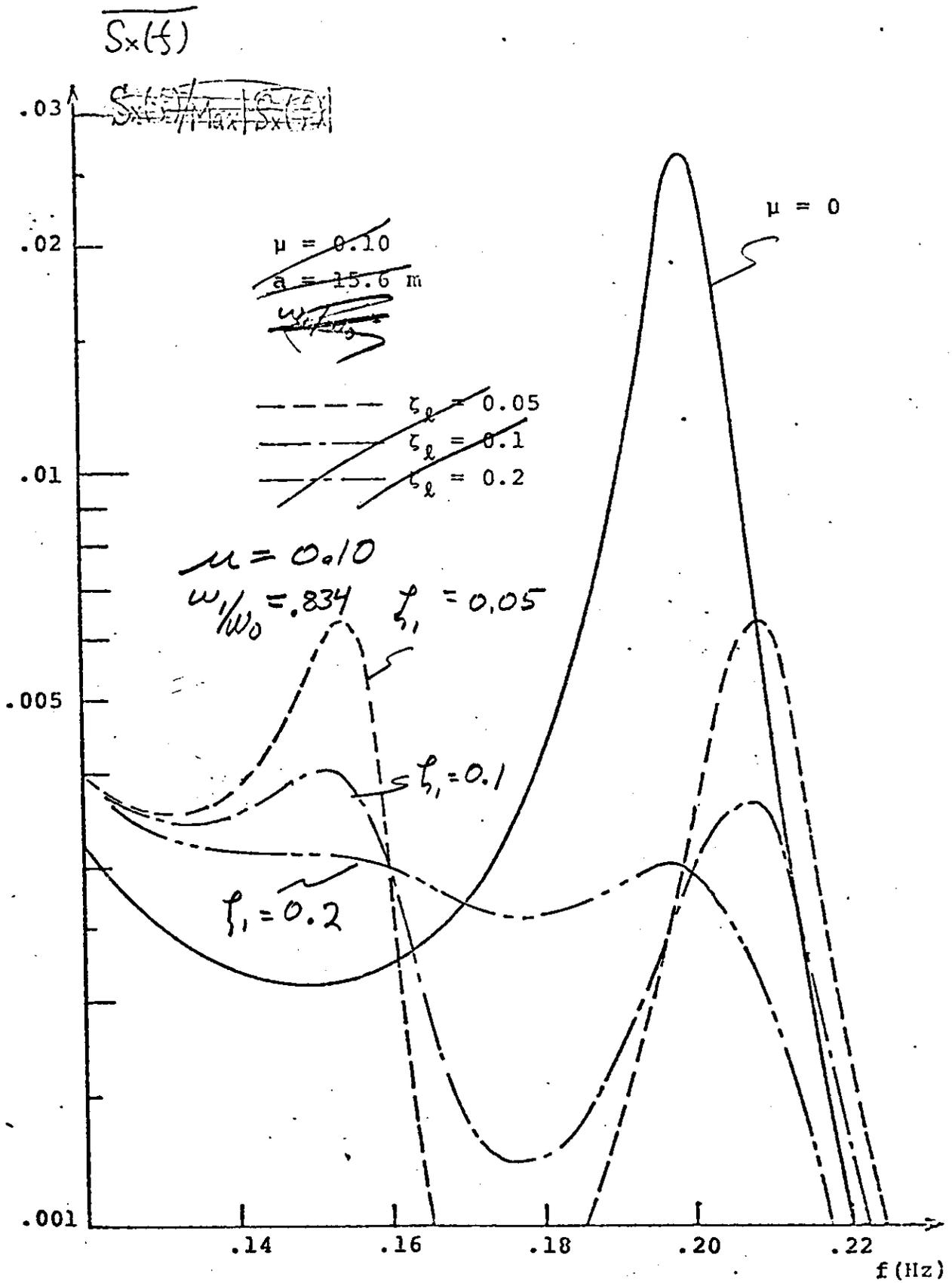
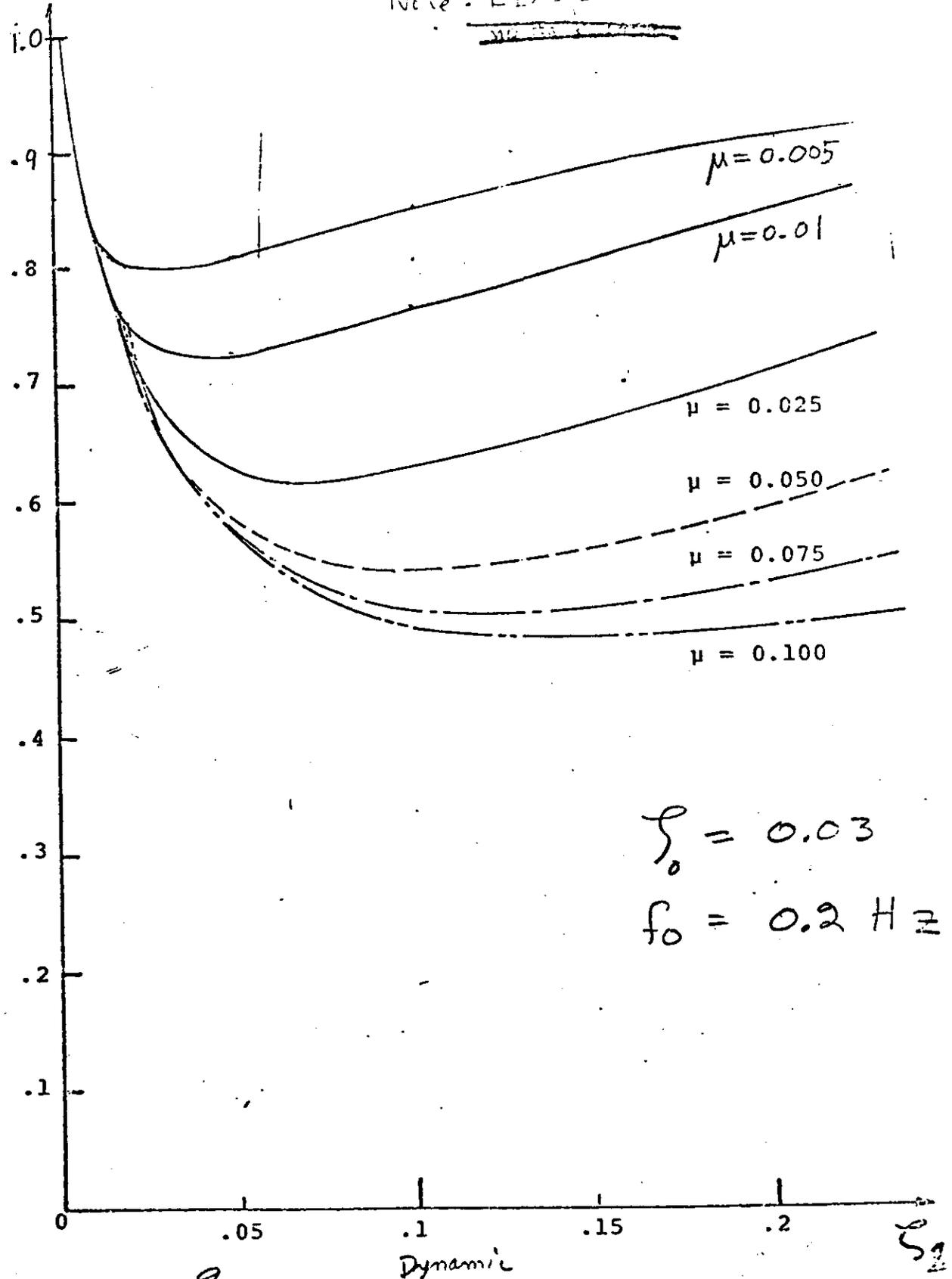


FIG. 8  RESPONSE DEFLECTION SPECTRUM OF 300 M PLATFORM  Example 2

$E[X_{\xi}^2]_{\text{dyna}} \mu=0$

Note:  $E[X_{\xi}^2]$  is based on ~~the following~~



$\xi_0 = 0.03$   
 $f_0 = 0.2 \text{ Hz}$

FIG. 9 MEAN SQUARE DEFLECTION OF 300 M PLATFORM